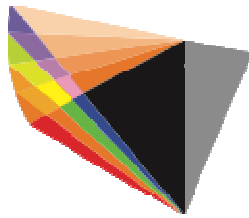


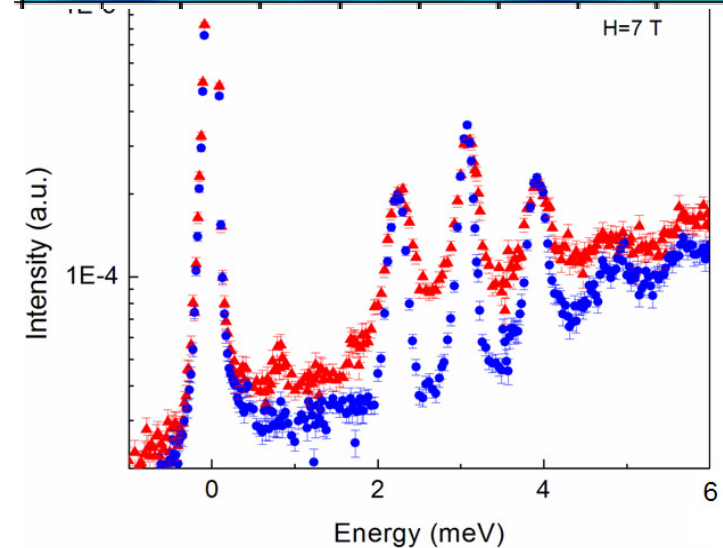
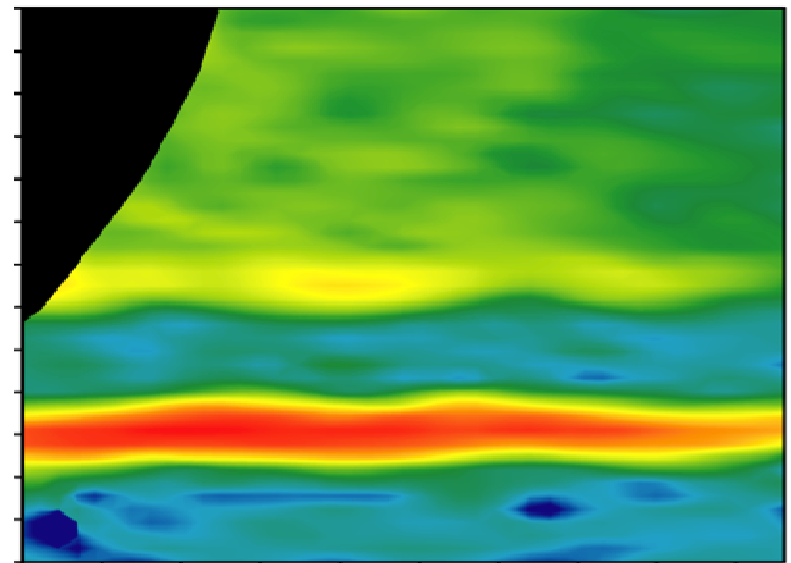
# Introduction to Triple Axis Neutron Spectroscopy

**Bruce D Gaulin**  
**McMaster University**



**Brockhouse Institute**  
for **Materials Research**

- *The triple axis spectrometer*
- *Constant-Q and constant E*
- *Practical concerns*
- *Resolution and Spurions*



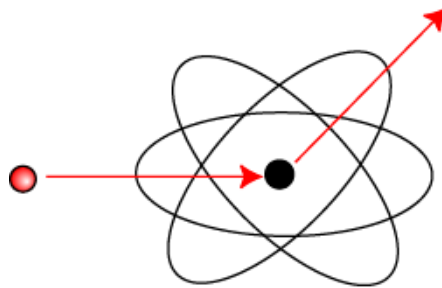
# Neutron interactions with matter

- **Properties of the neutron**

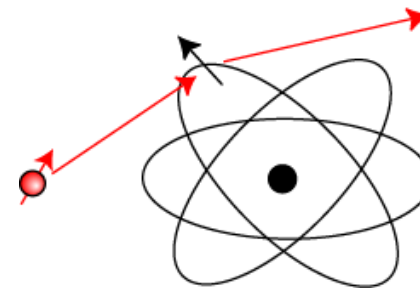
- Mass  $m_n = 1.675 \times 10^{-27}$  kg
- Charge 0
- Spin-1/2, magnetic moment  $\mu_n = -1.913 \mu_N$

- **Neutrons interact with...**

- Nucleus
- Crystal structure/excitations (eg. Phonons)
- Unpaired electrons via dipole scattering
- Magnetic structure and excitations



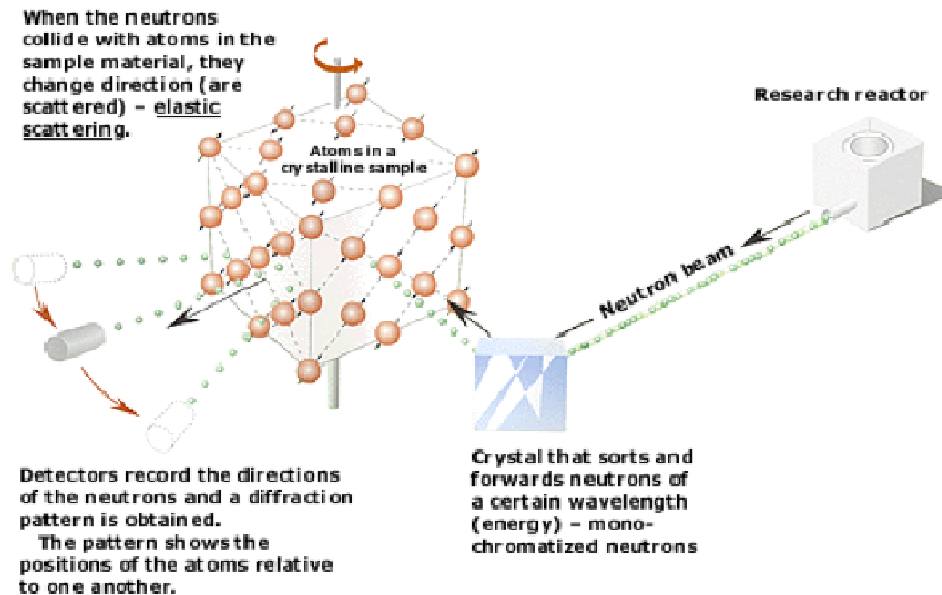
**Nuclear scattering**



**Magnetic dipole scattering**

# Brockhouse and Shull Share 1994 Nobel Prize in Physics

Most recent Nobel Prize awarded to a scientist working in Canada

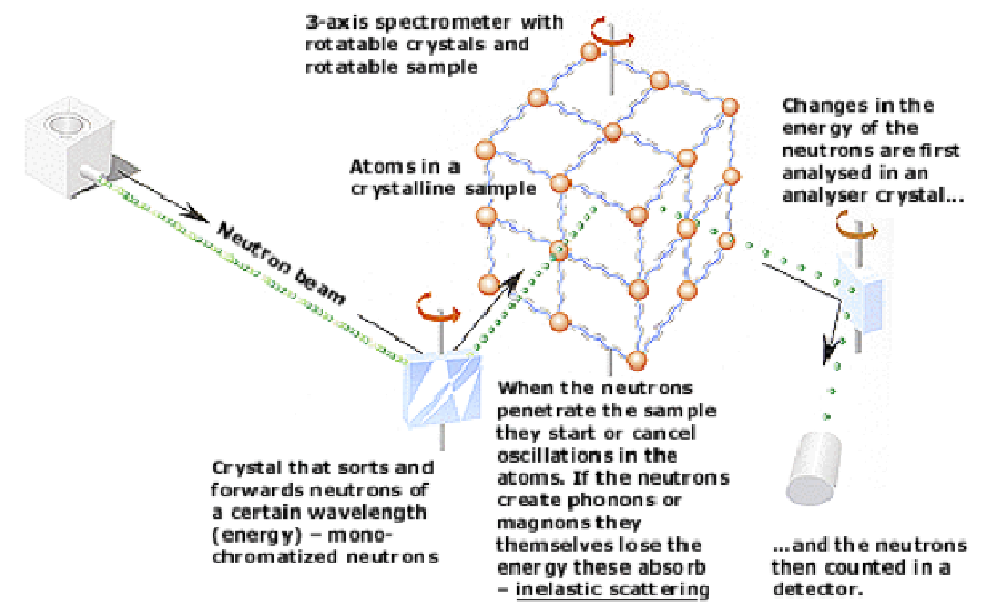


**S**

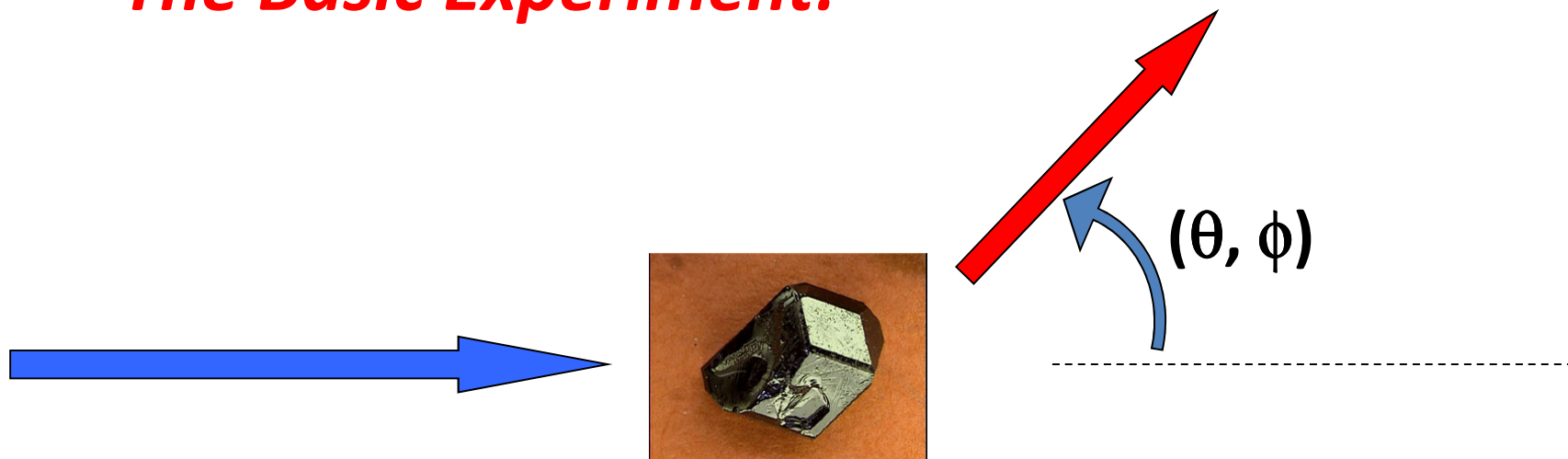
**Clifford G. Shull**, MIT, Cambridge, Massachusetts, USA, receives one half of the 1994 Nobel Prize in Physics for development of the neutron diffraction technique.

**B**

**Bertram N. Brockhouse**, McMaster University, Hamilton, Ontario, Canada, receives one half of the 1994 Nobel Prize in Physics for the development of neutron spectroscopy.



## *The Basic Experiment:*



### **Incident Beam:**

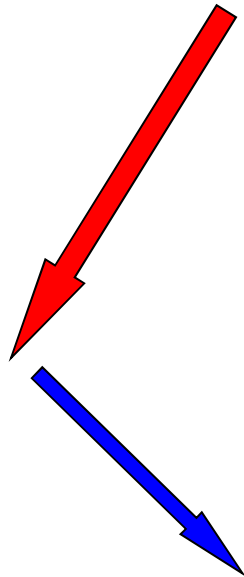
- monochromatic
- “white”
- “pink”

### **Scattered Beam:**

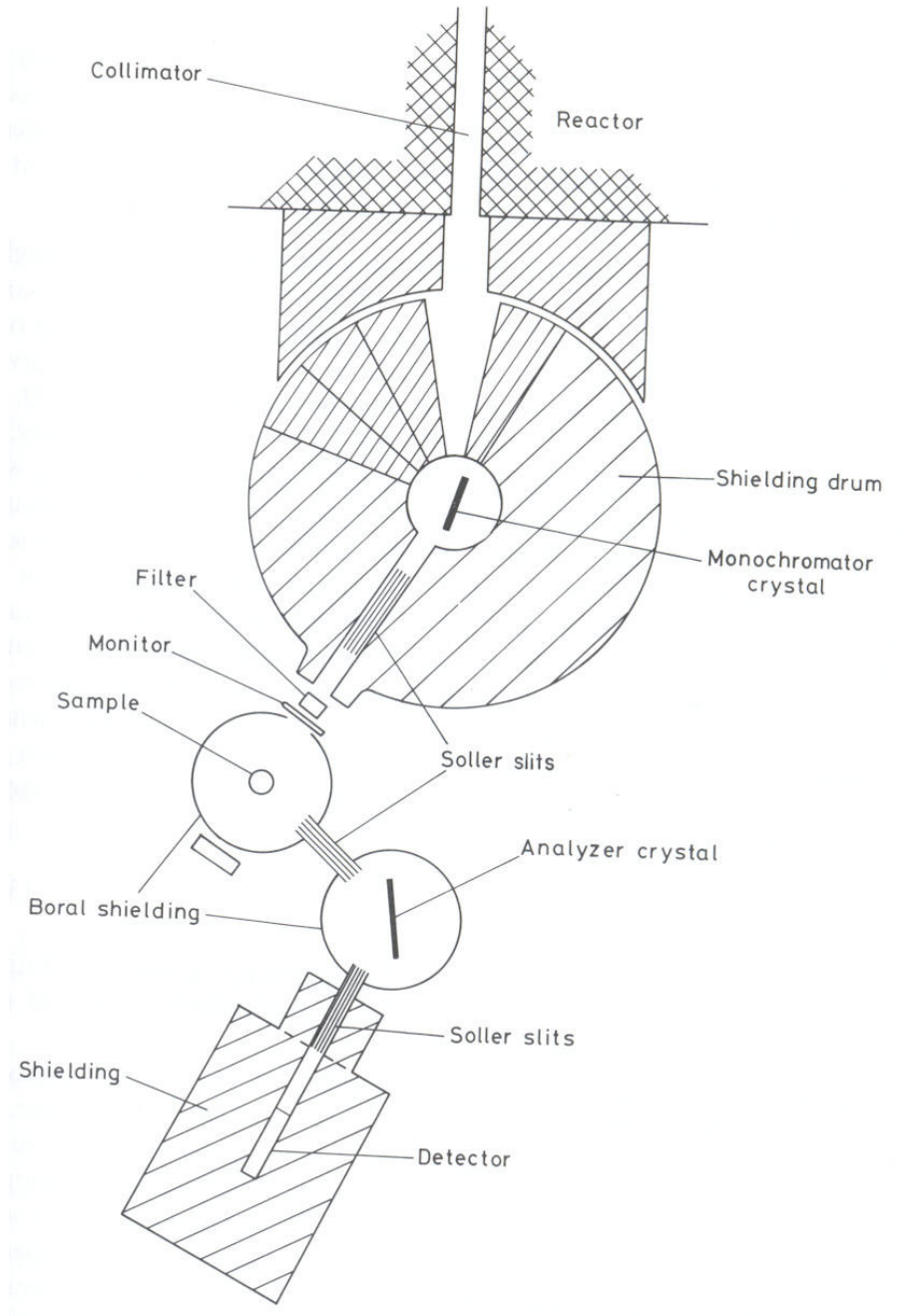
- Resolve its energy
- Don't resolve its energy
- Filter its energy

# Brockhouse's Triple Axis Spectrometer

$$| \mathbf{k}_i | = 2 \pi / \lambda_i$$

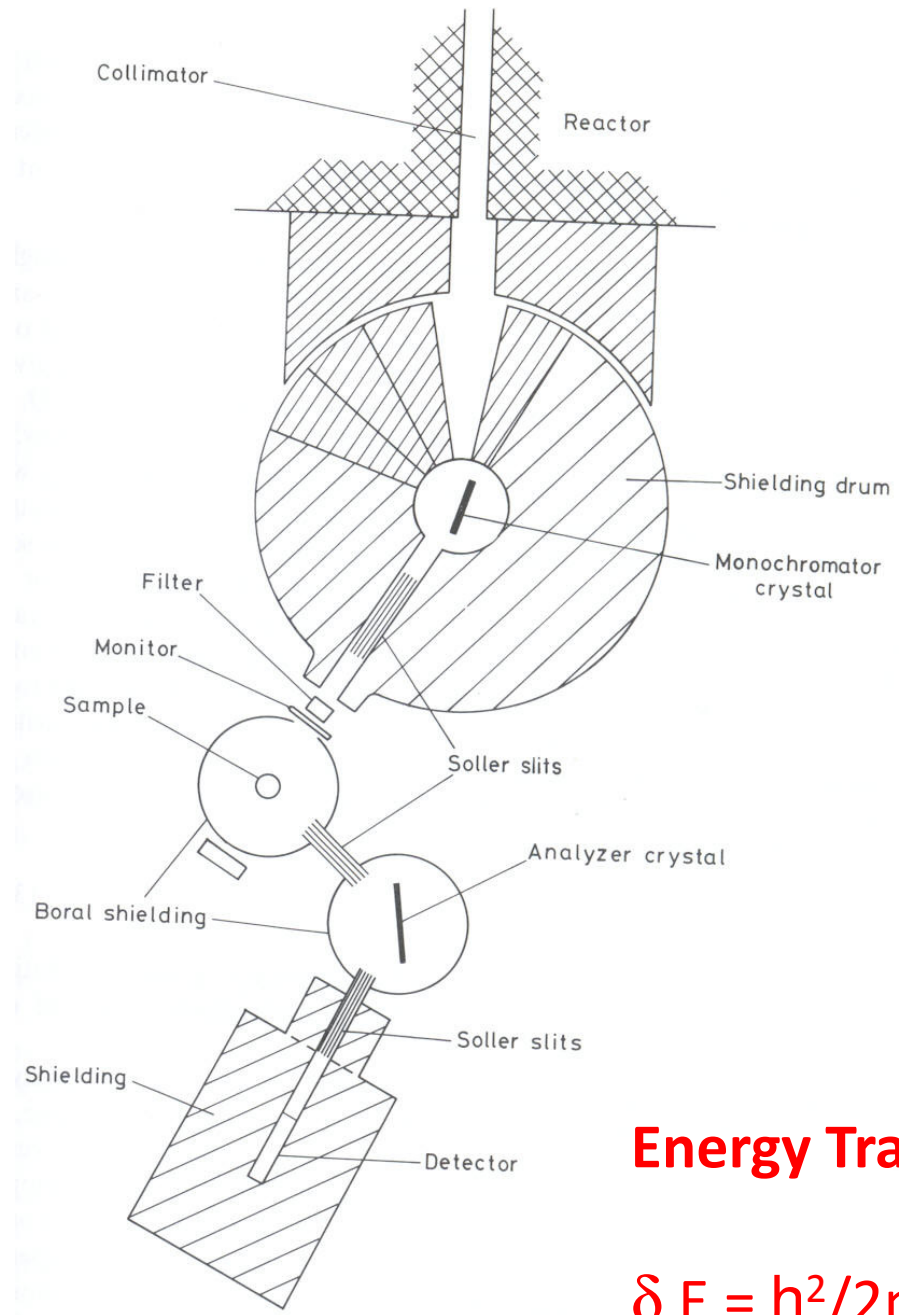
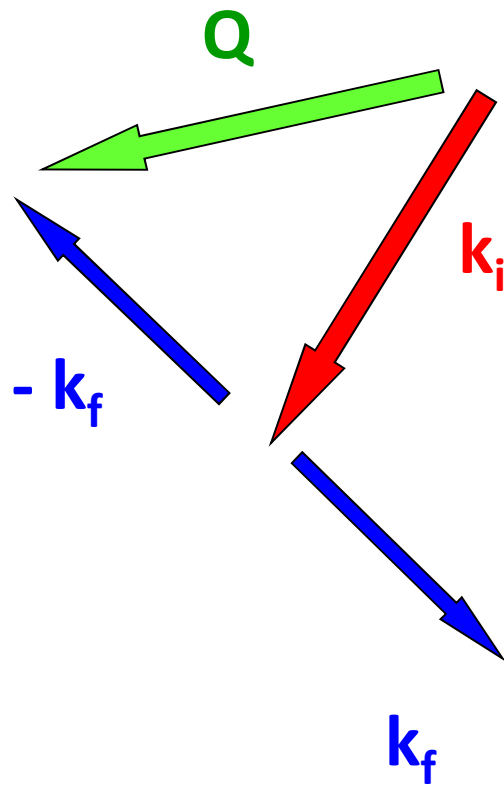


$$| \mathbf{k}_f | = 2 \pi / \lambda_f$$



# Momentum Transfer:

$$Q = k_i - k_f$$



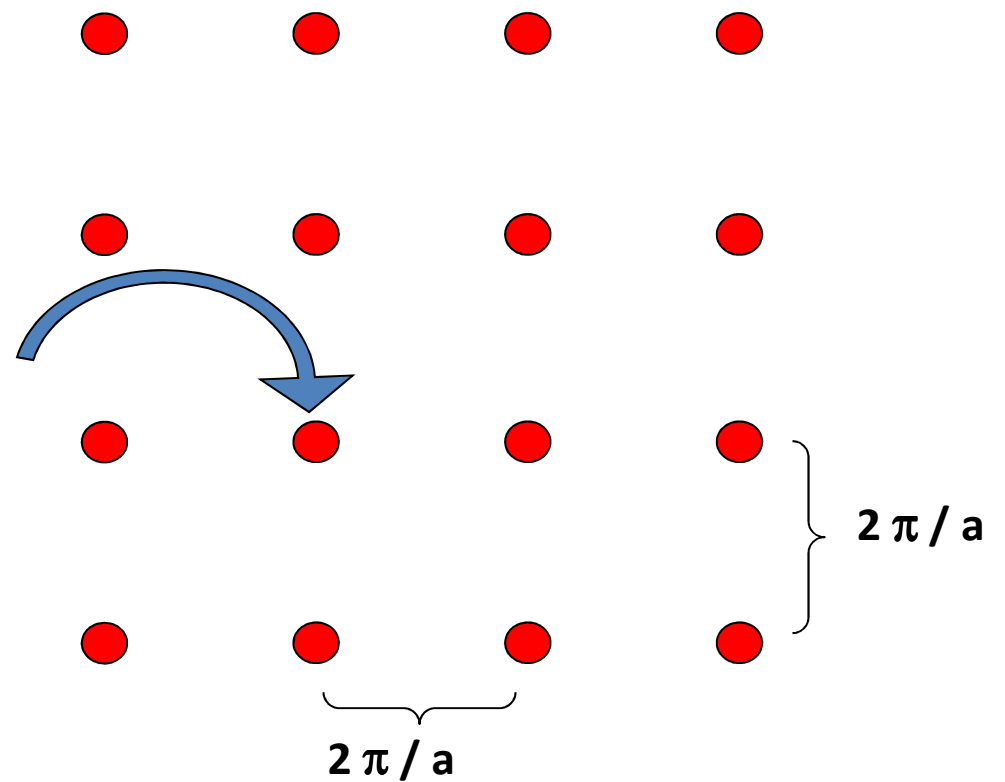
# Energy Transfer:

$$\delta E = \frac{h^2}{2m} (k_i^2 - k_f^2)$$

# Mapping Momentum – Energy (Q-E) space

Origin of reciprocal space;

Remains fixed for any sample rotation

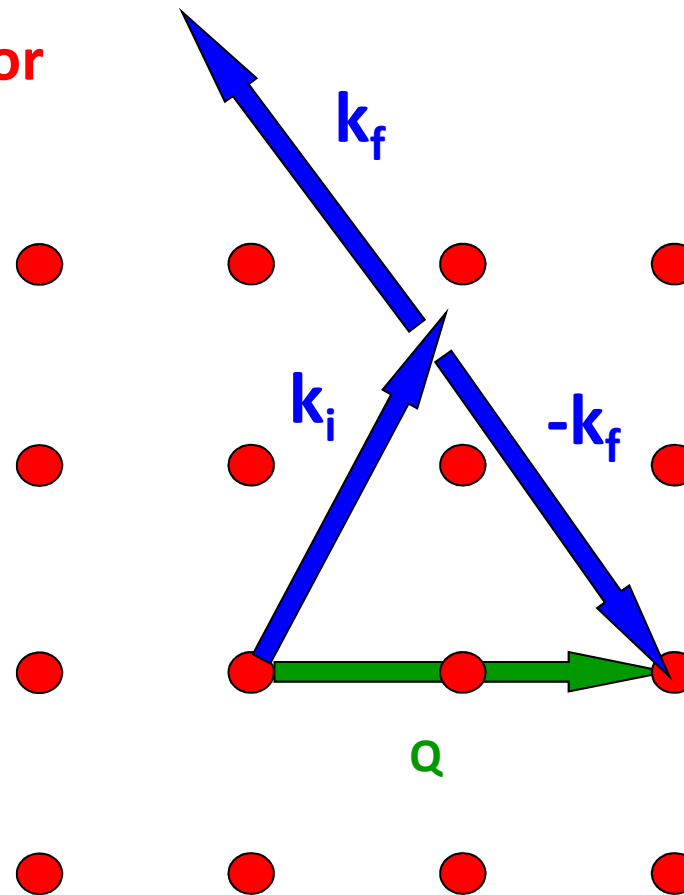


**Bragg diffraction:**

**Constructive Interference**

**Q = Reciprocal Lattice Vector**

**Elastic scattering :  $|k_i| = |k_f|$**

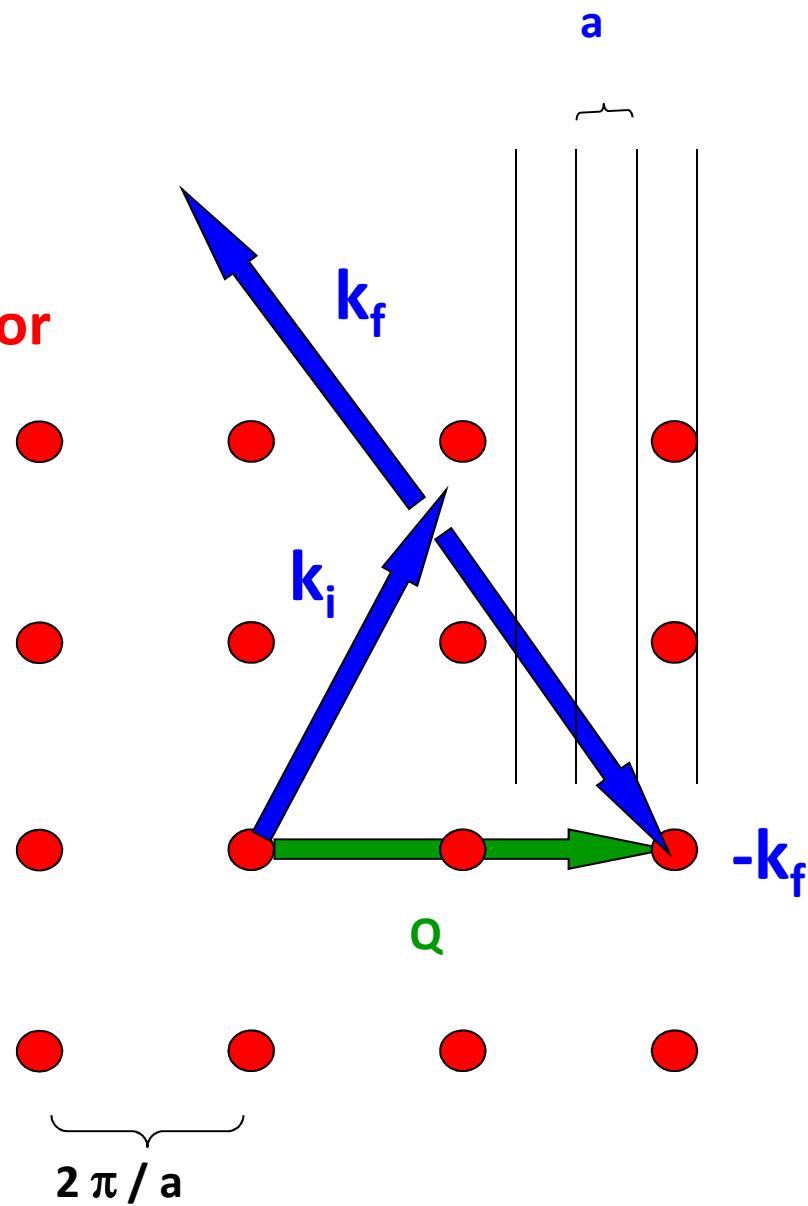




**Bragg diffraction:**

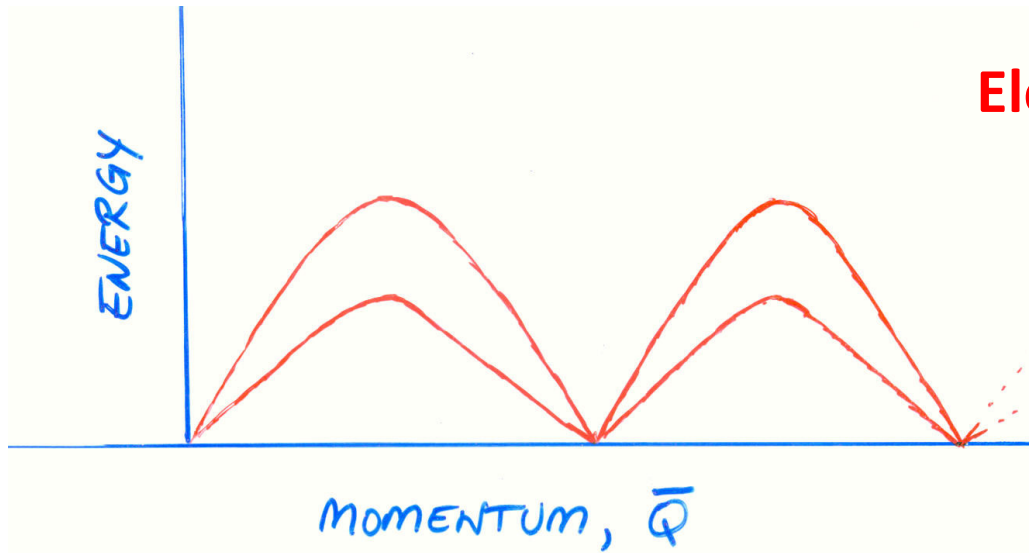
**Constructive Interference**

**Q = Reciprocal Lattice Vector**

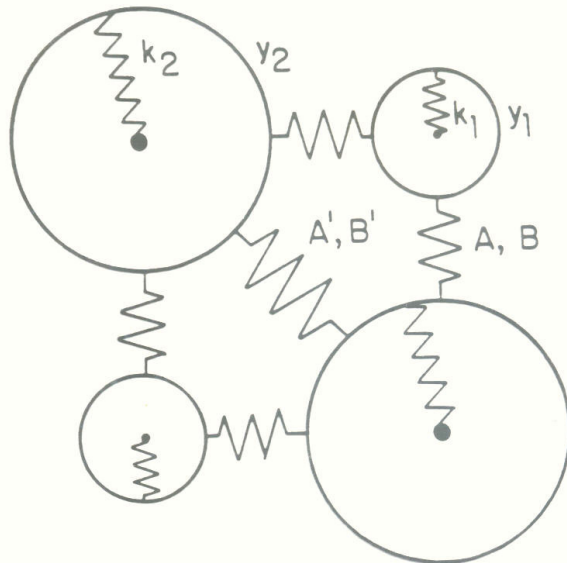


**Elastic scattering :  $|k_i| = |k_f|$**

## Elementary Excitations in Solids

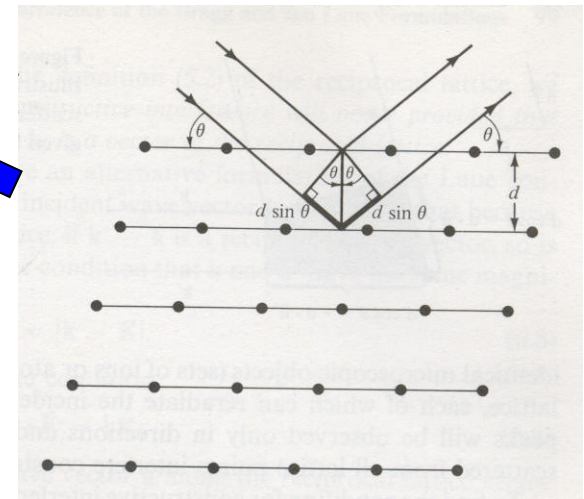
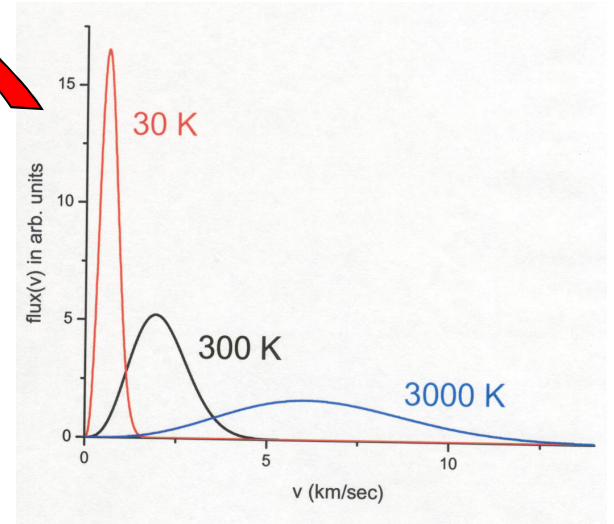
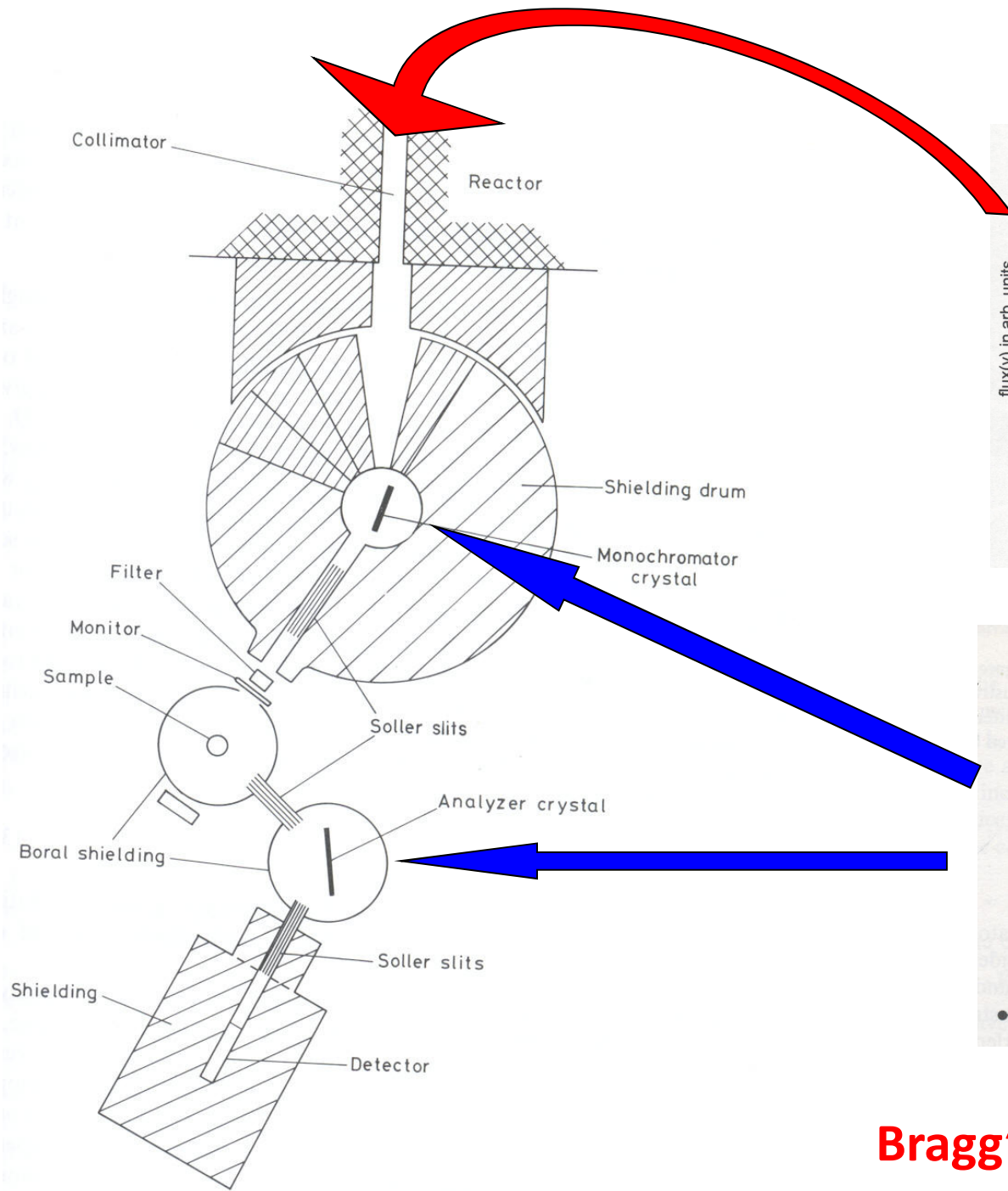


- Lattice Vibrations (Phonons)
- Spin Fluctuations (Magnons)

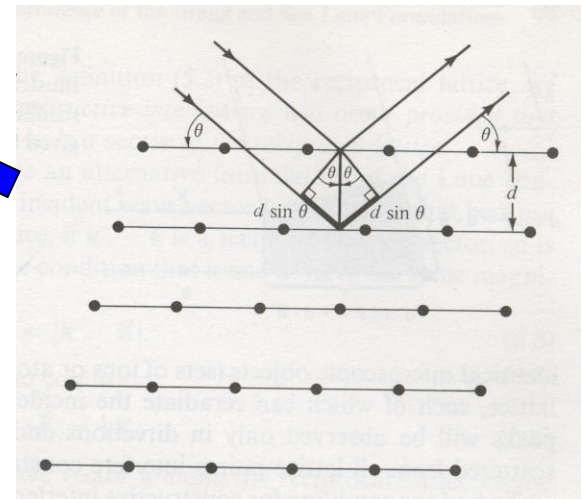
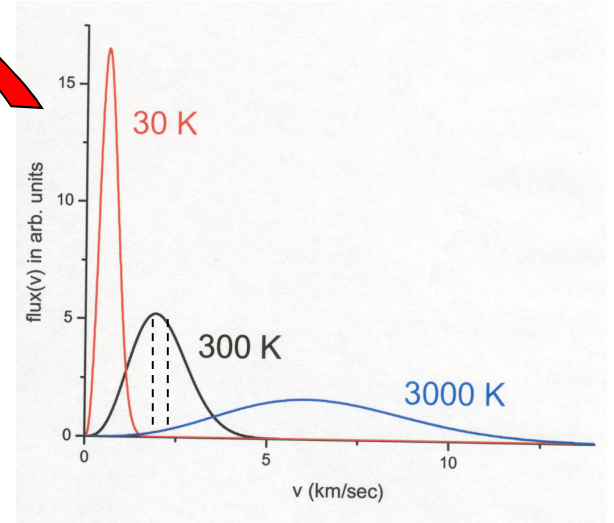
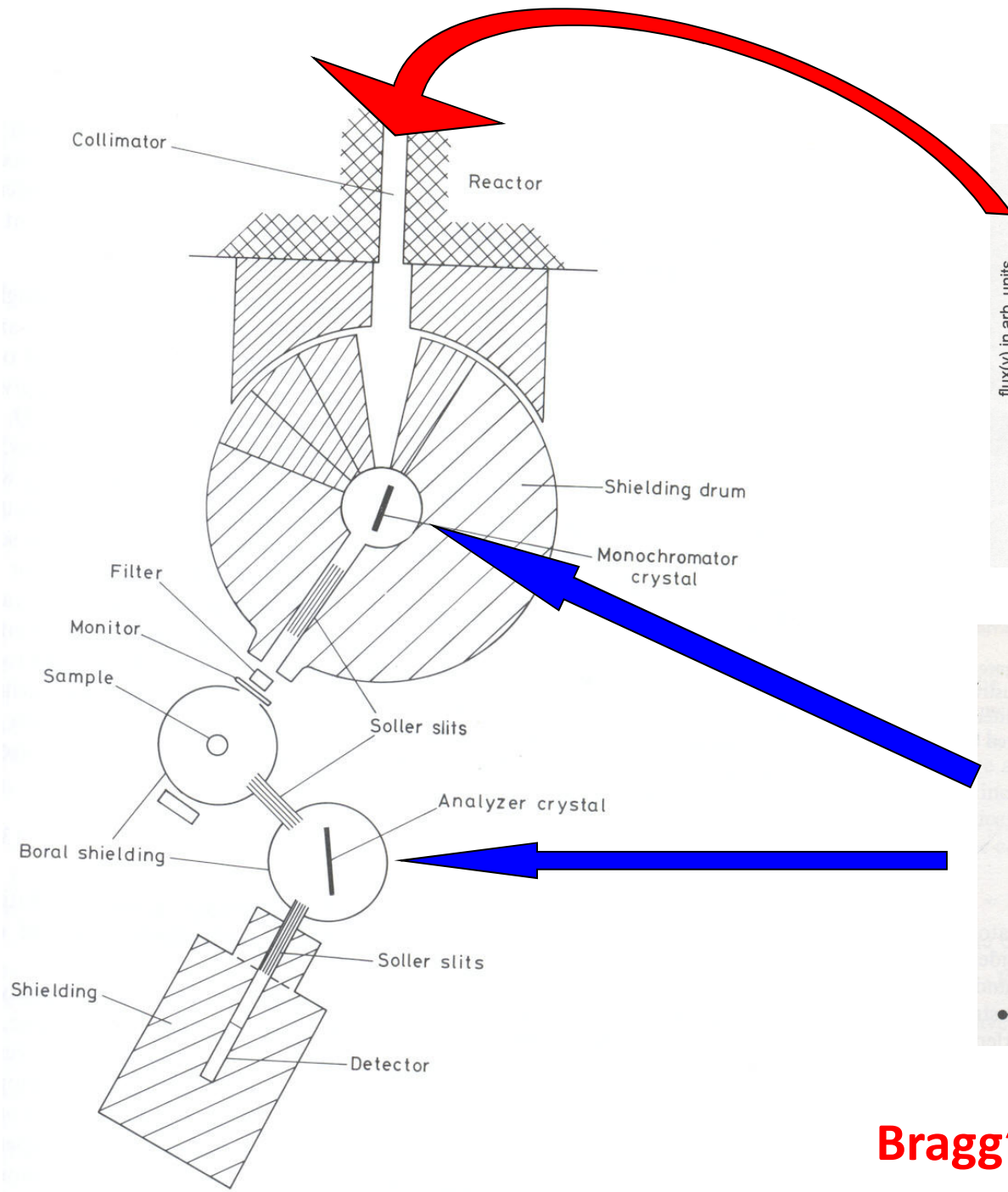


## Energy vs Momentum

- Forces which bind atoms together in solids



**Bragg's Law:  $n\lambda = 2d \sin(\theta)$**

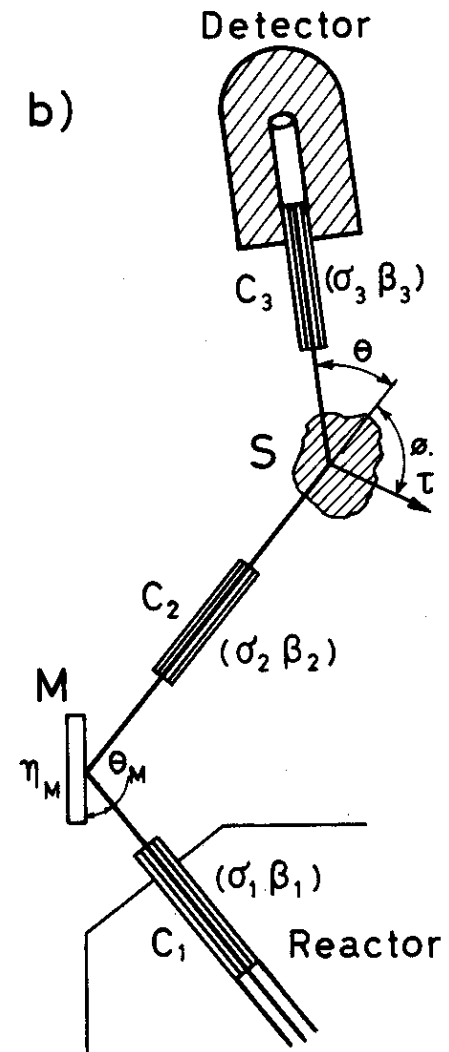
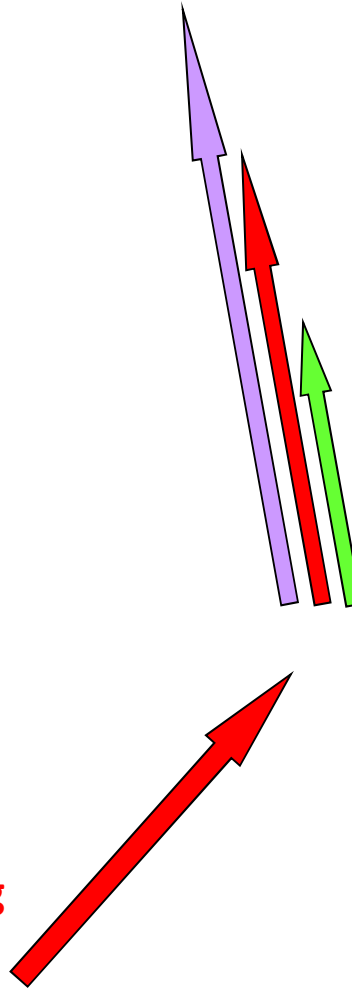


**Bragg's Law:  $n\lambda = 2d \sin(\theta)$**

## Two Axis Spectrometer:

- 3-axis with analyser removed
- Powder diffractometer
- Small angle diffractometer
- Reflectometers

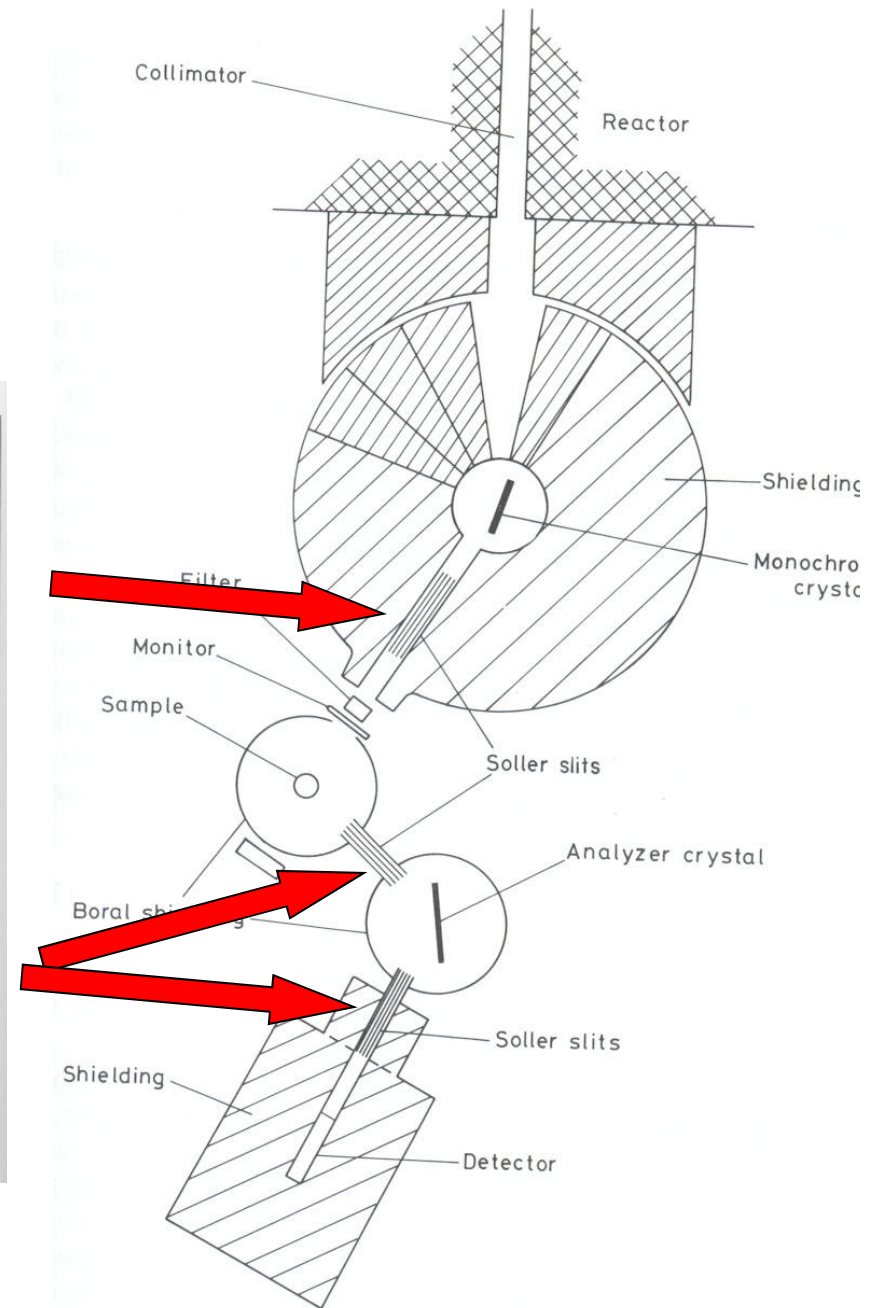
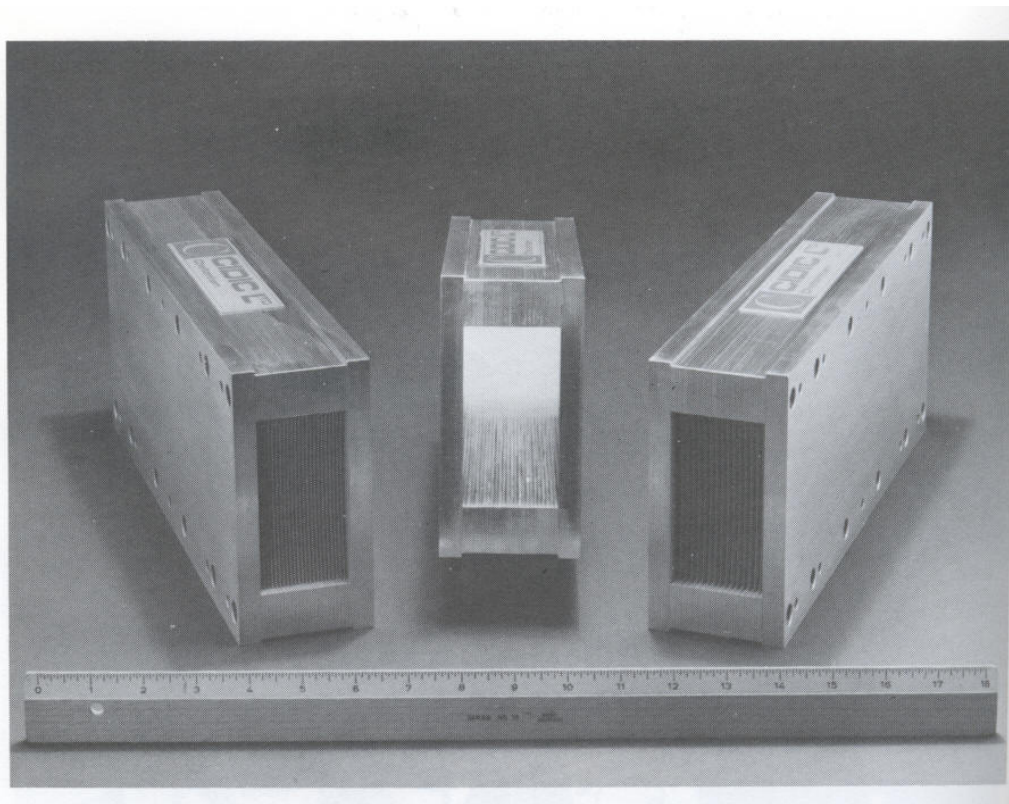
Diffractometers often employ working assumption that all scattering is *elastic*.





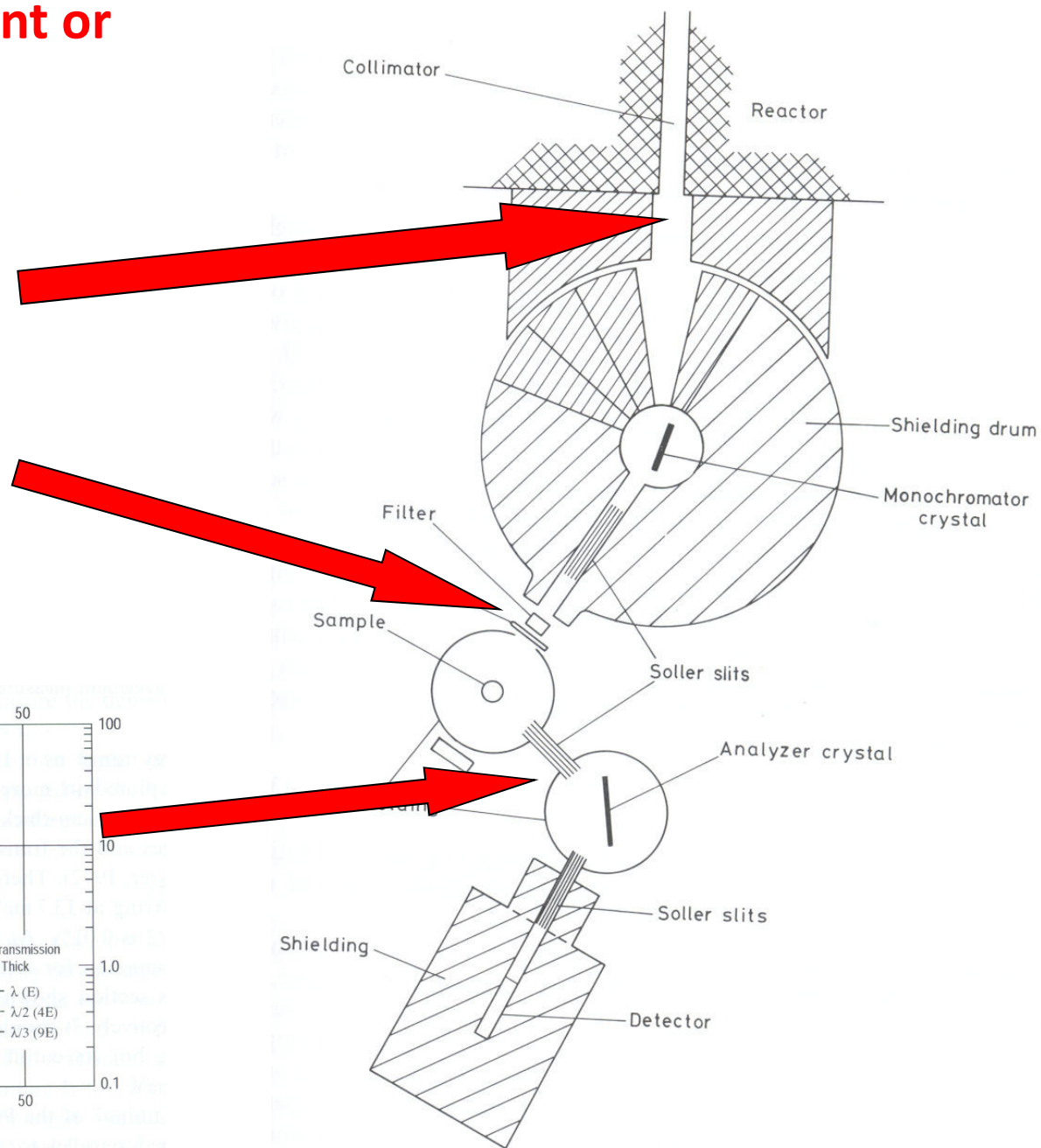
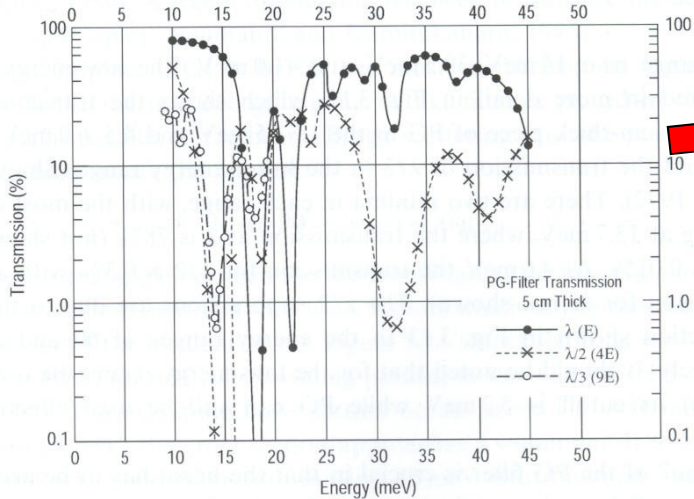
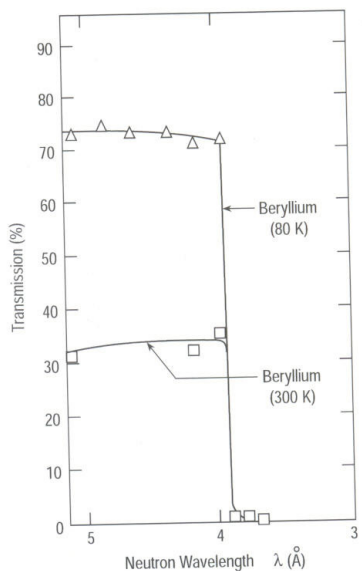
## Soller Slits: Collimators

Define beam direction to  
 $\pm 0.5, 0.75$  etc. degrees



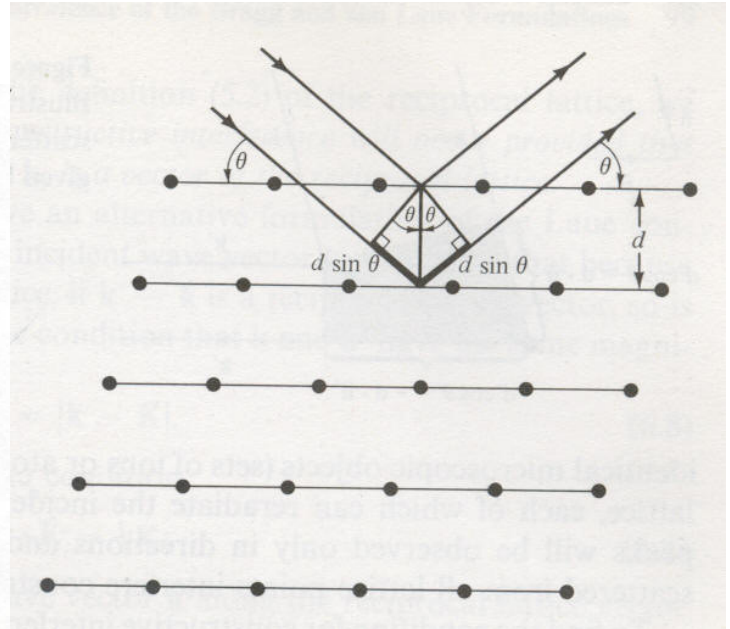
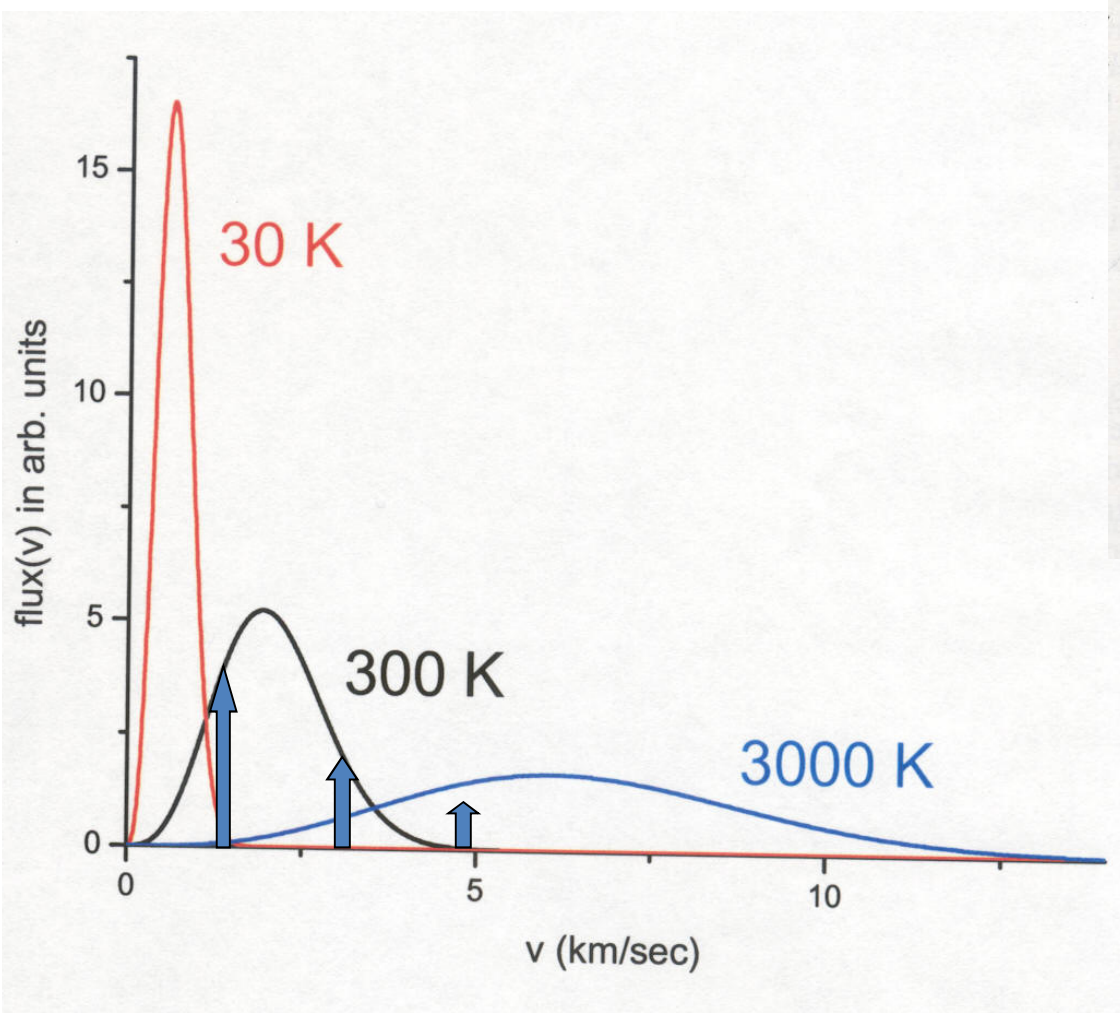
# Filters:

Remove  $\lambda/n$  from incident or scattered beam, or both



# Single crystal monochromators:

## Bragg reflection and harmonic contamination

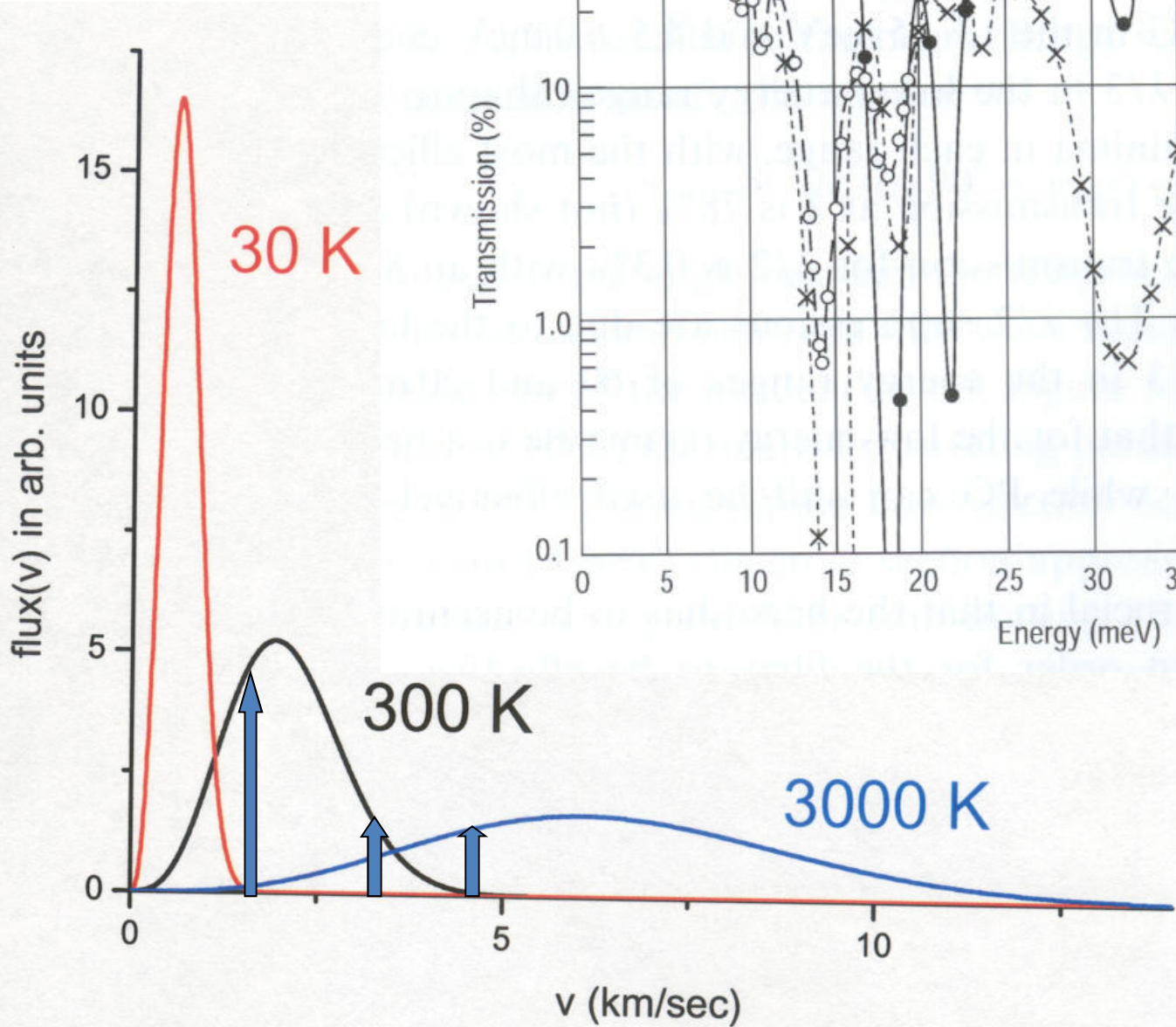


$$n\lambda = 2d \sin(\theta)$$

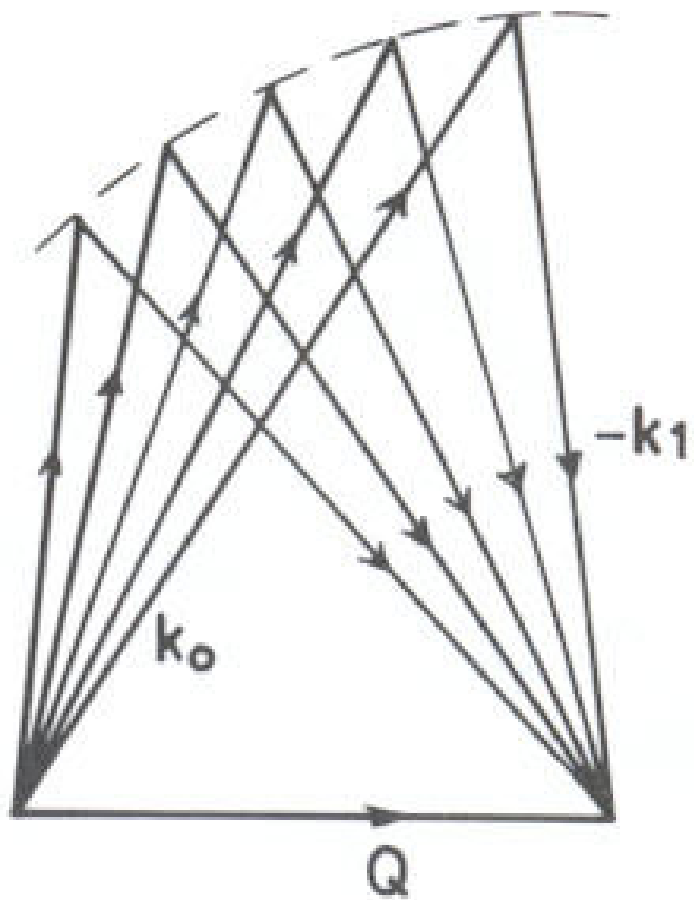
Get:  $\lambda, \lambda/2, \lambda/3, \text{etc.}$



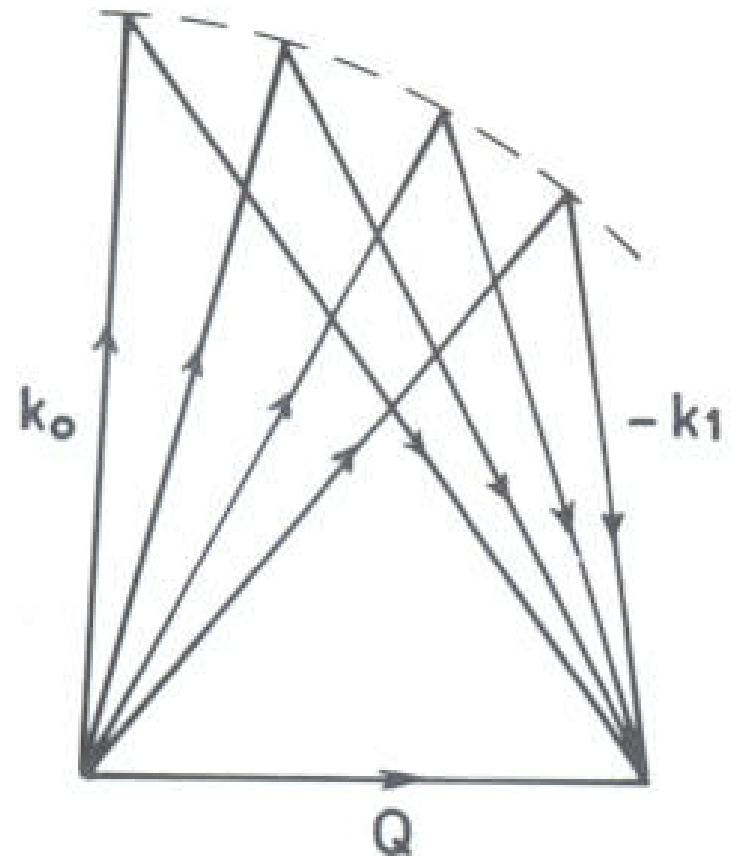
**Pyrolytic graphite filter:**



$E = 14.7 \text{ meV}$   
 $\lambda = 2.37 \text{ \AA}$   
 $v = 1.6 \text{ km/s}$   
 $2 \times v = 3.2 \text{ km/s}$   
 $3 \times v = 4.8 \text{ km/s}$



Constant  $k_f$  (ii)



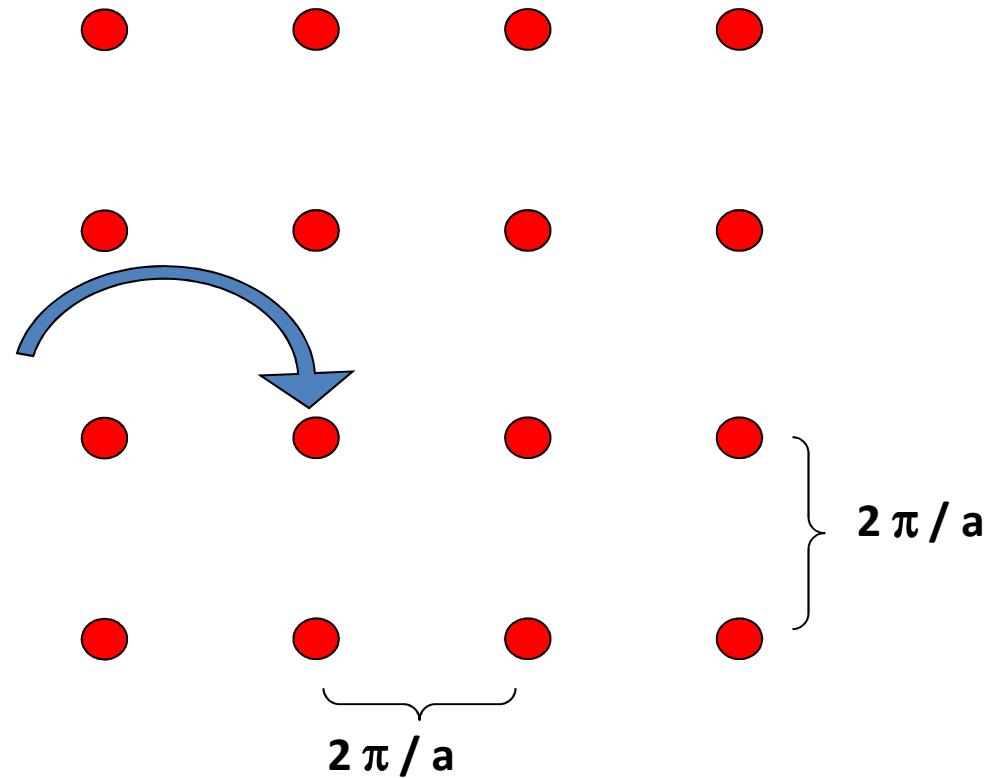
Constant  $k_i$  (i)

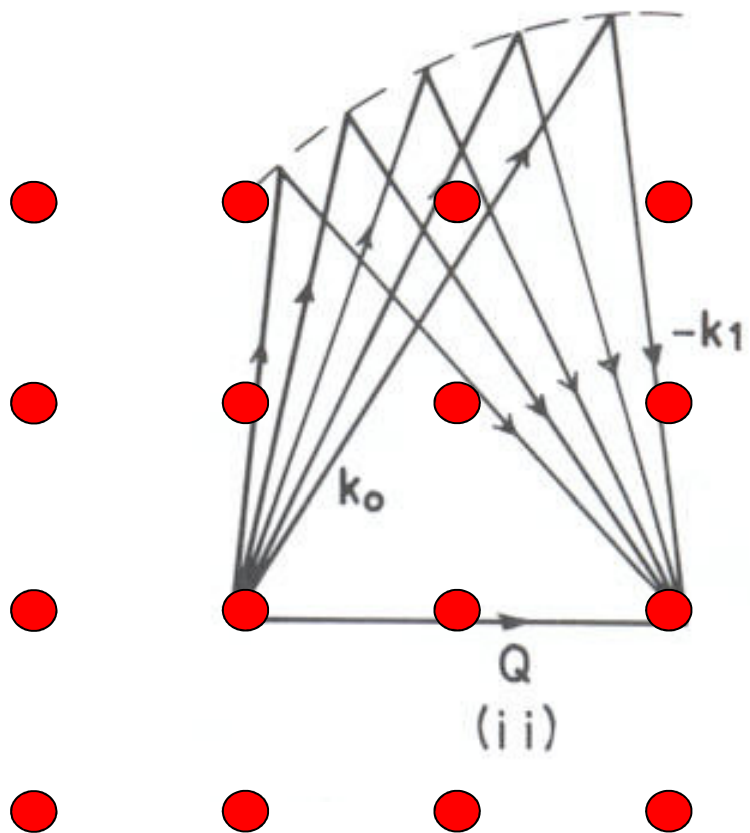
Two different ways of performing constant-Q scans

# Mapping Momentum – Energy (Q-E) space

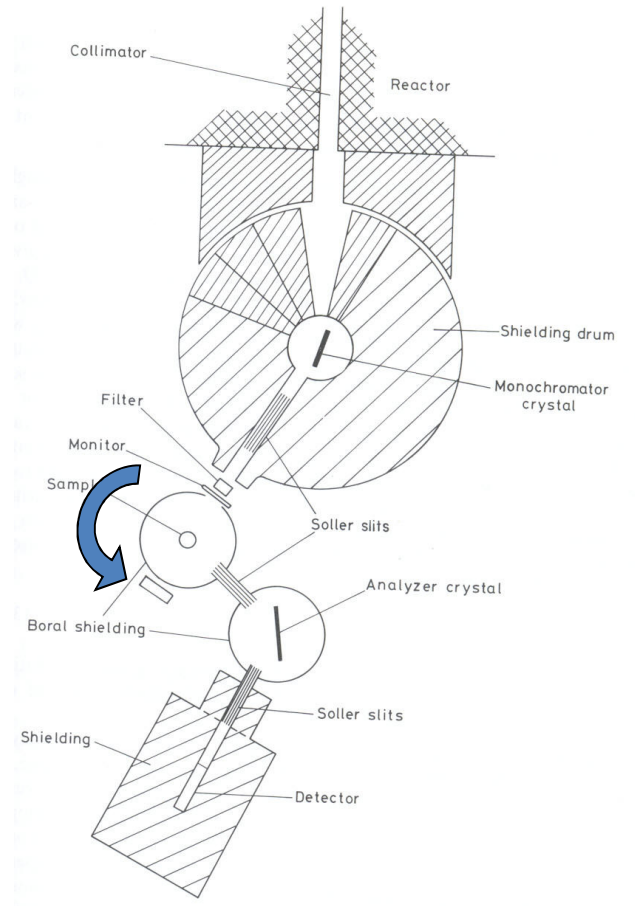
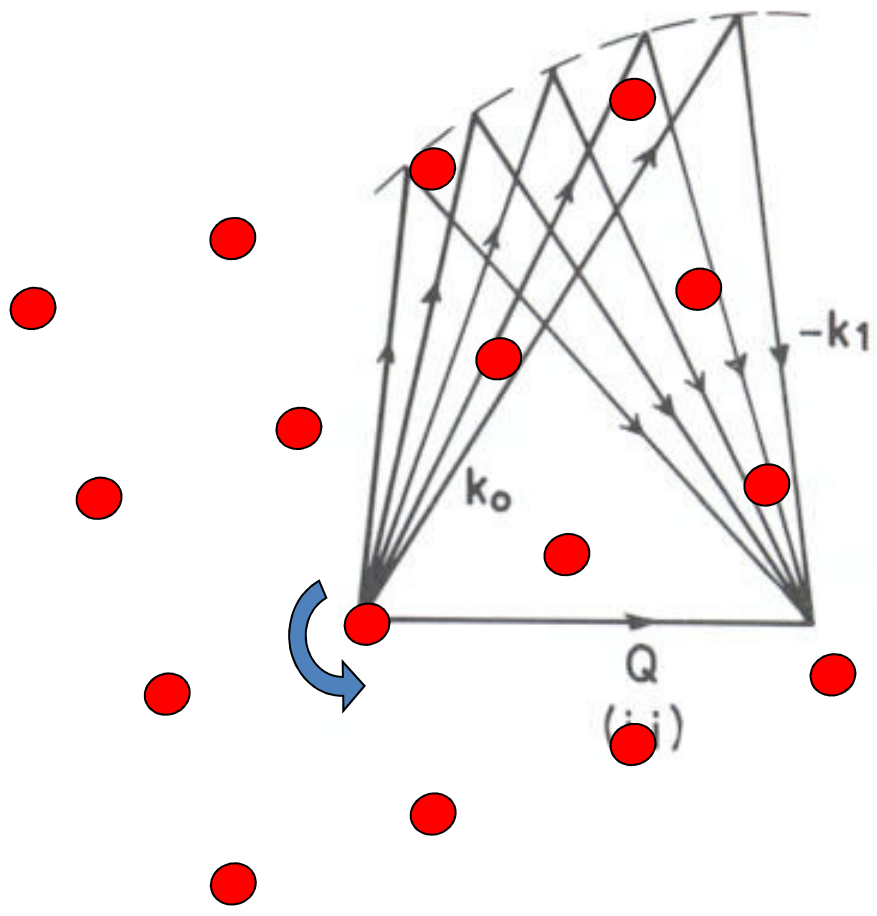
Origin of reciprocal space;

Remains fixed for any sample rotation





(ii)



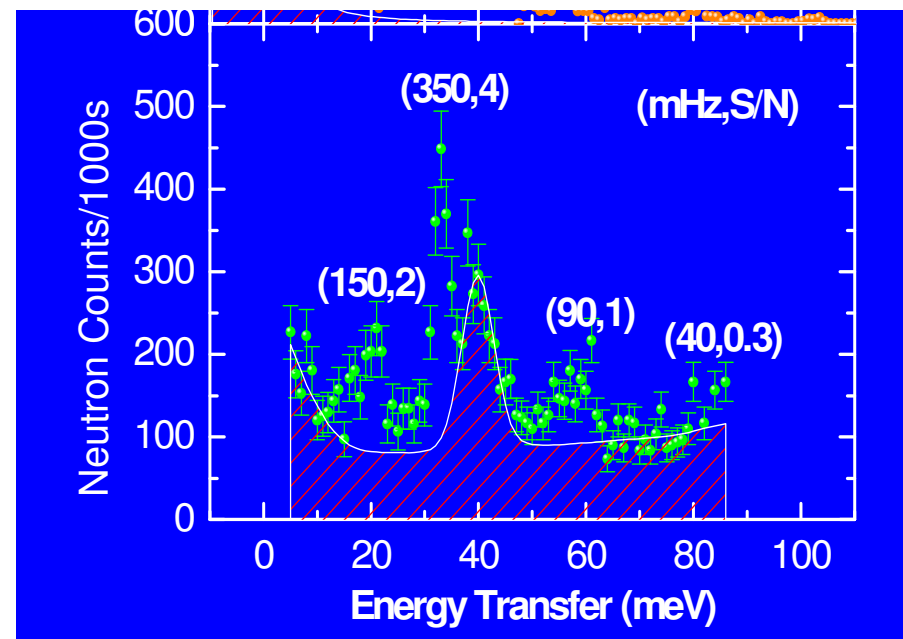
# Spurions

- **Bragg – incoherent – Bragg**

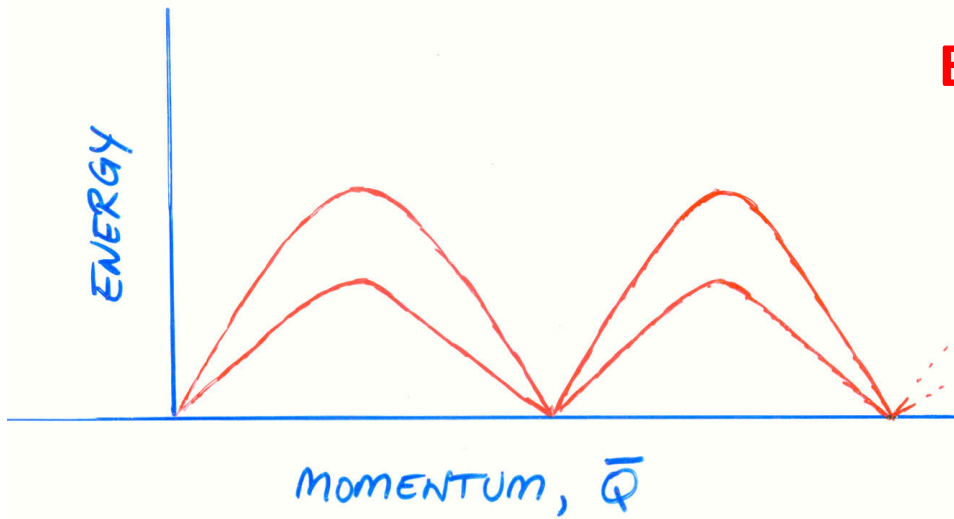
- Eg.  $k_i = 2k_f$ 
  - $\hbar\omega = 41.1$  meV
  - $E_f = 13.7$  meV
  - $E_i = 54.8$  meV
  - $4E_f = 54.8$  meV
  - Incoherent elastic scattering visible from analyzer  $\lambda/2$

- **incoherent – Bragg – Bragg**

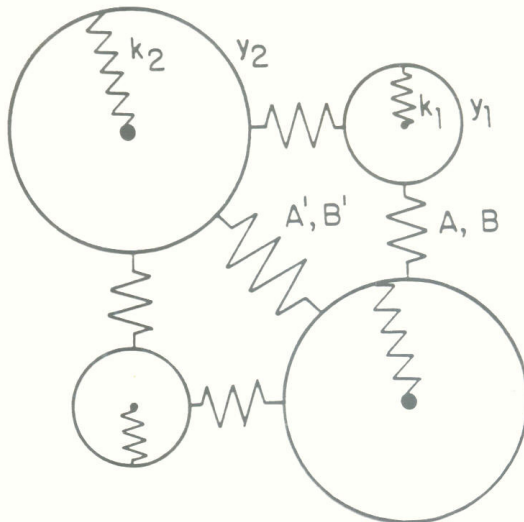
- Sample  $2\theta$  in Bragg condition for  $k_f - k_i$
- Even for inelastic config, weak incoherent from mono



## Elementary Excitations in Solids

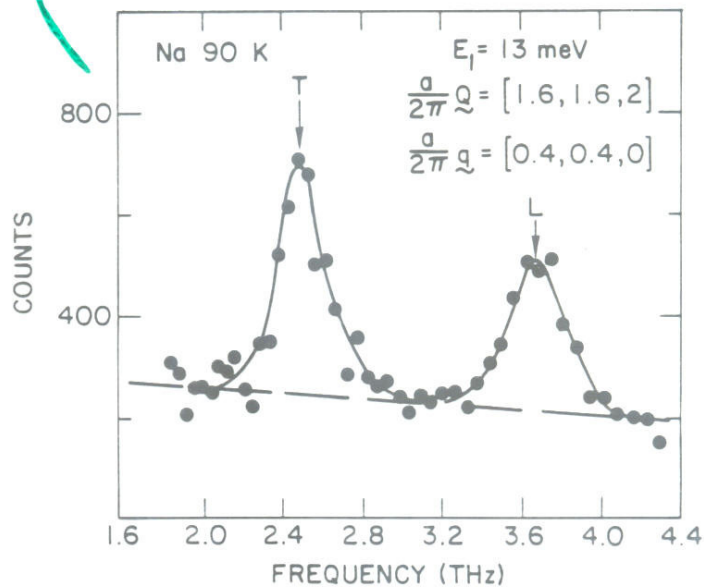
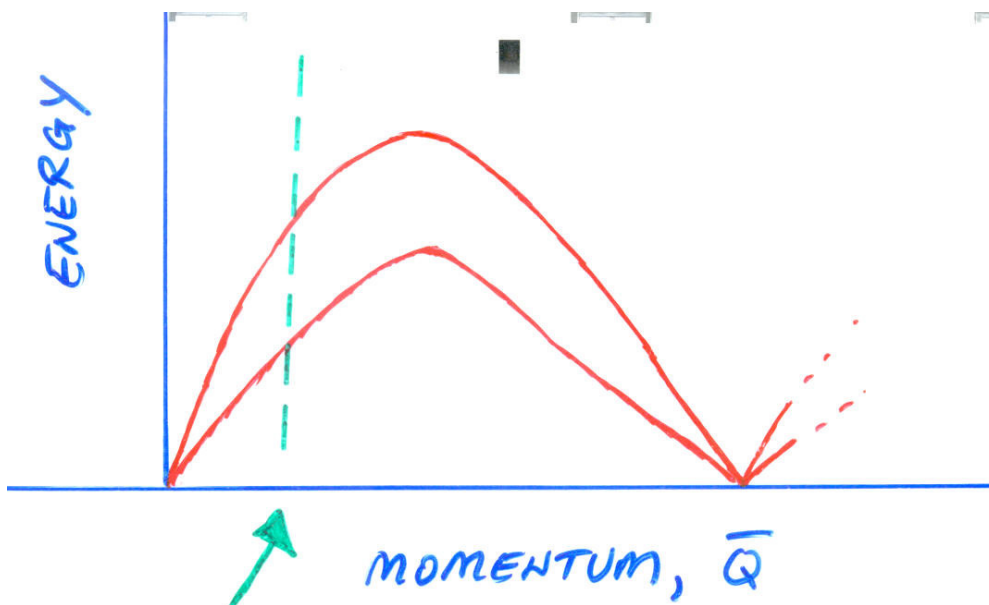


- Lattice Vibrations (Phonons)
- Spin Fluctuations (Magnons)



## Energy vs Momentum

- Forces which bind atoms together in solids



**Constant Q, Constant E**  
**3-axis technique allow us to**  
**Put Q-Energy space on a grid,**  
**And scan through as we wish**

**Map out elementary excitations**  
**In Q-energy space (dispersion**  
**Surface)**



# Phonons

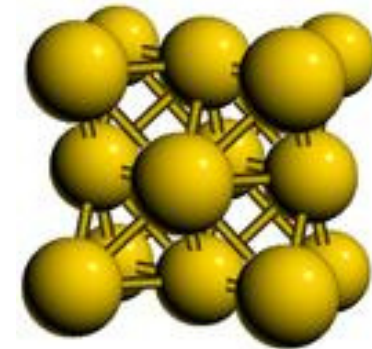
- Normal modes in periodic crystal  $\rightarrow$  wavevector

$$\mathbf{u}(l,t) = \frac{1}{\sqrt{NM}} \sum_{j\mathbf{q}} \boldsymbol{\varepsilon}_j(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{l}) \hat{B}(\mathbf{q}j,t)$$

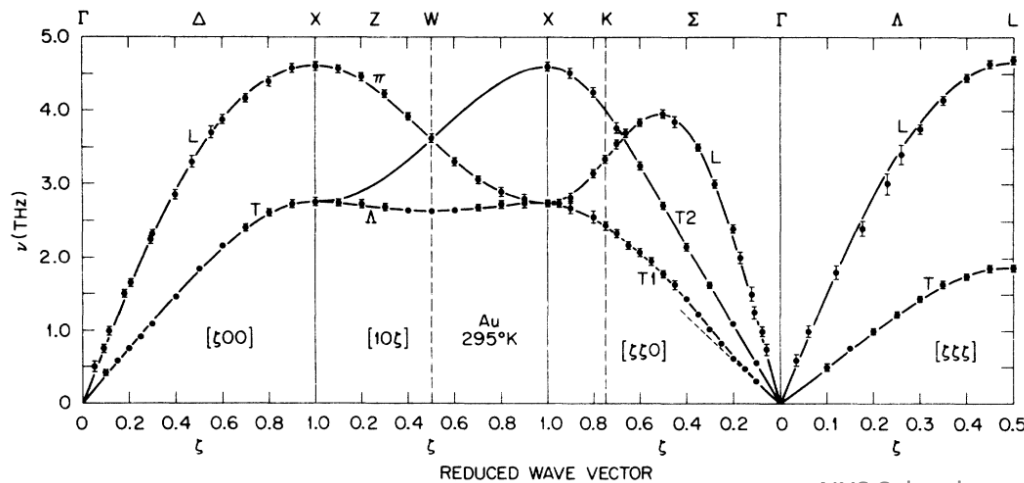
- Energy of phonon depends on  $\mathbf{q}$  and polarization



Longitudinal mode

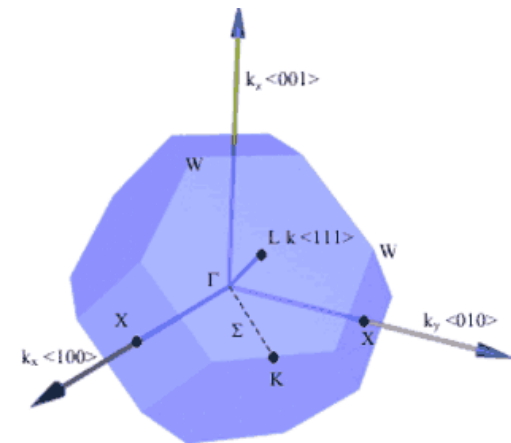


FCC structure



Lynn, et al., *Phys. Rev. B* **8**, 3493 (1973).

NXS School

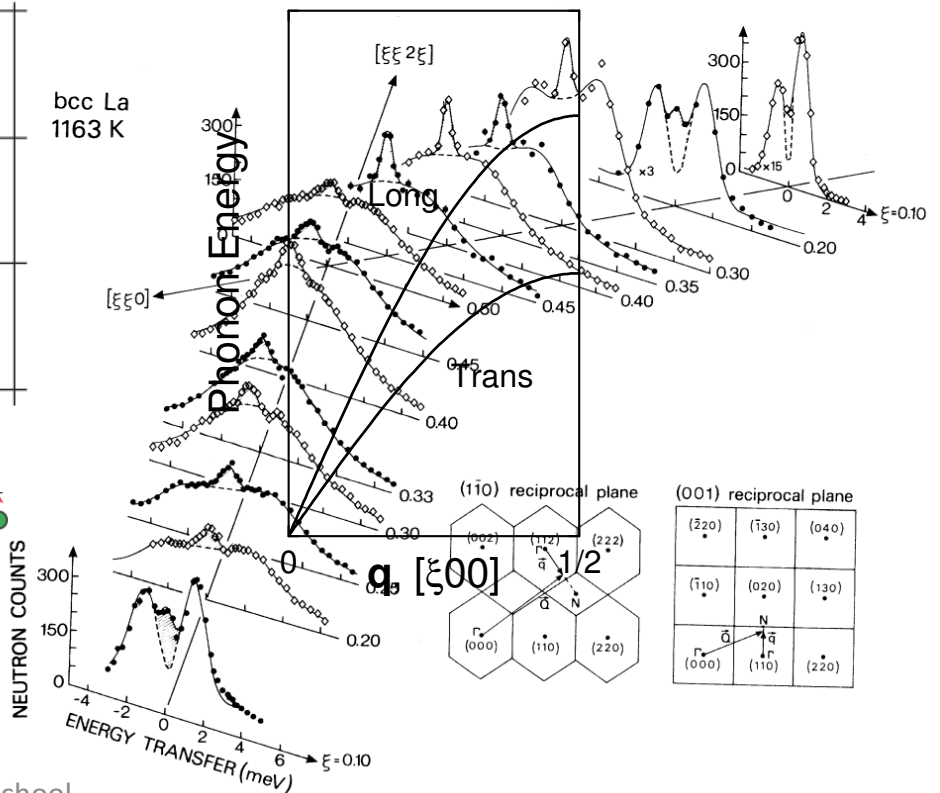
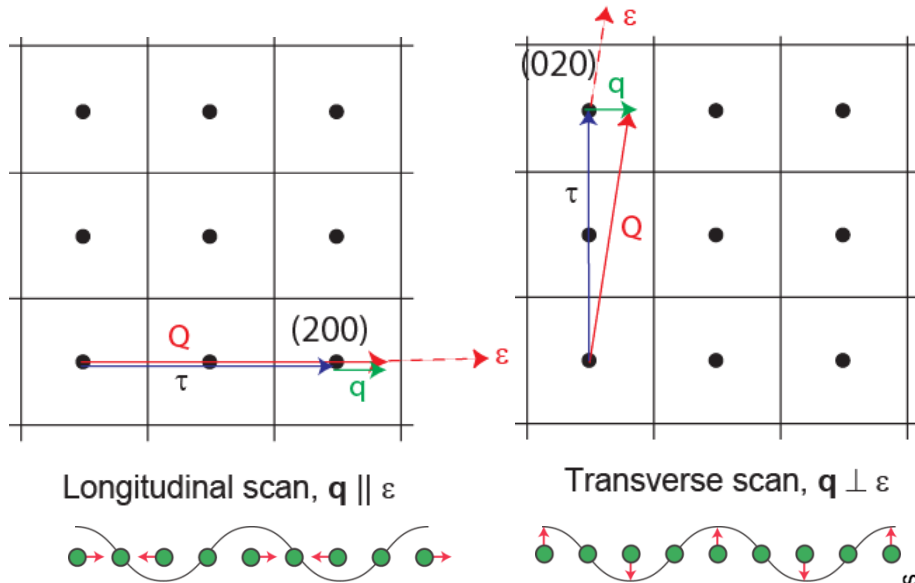


FCC Brillouin zone

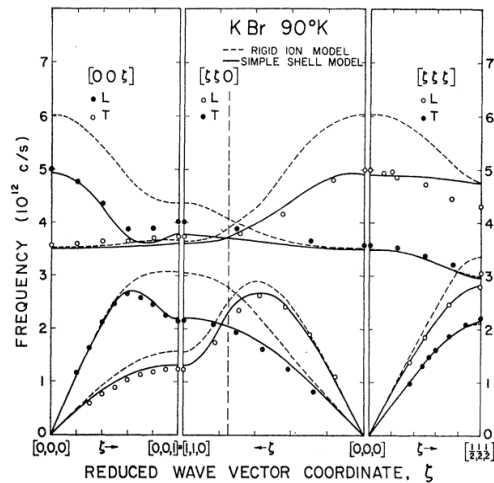
# Phonon intensities

$$S_{1+}(\mathbf{Q}, \omega) = \frac{1}{2NM} e^{-Q^2 \langle u^2 \rangle} \sum_{j\mathbf{q}} \frac{|\mathbf{Q} \cdot \boldsymbol{\varepsilon}_j(\mathbf{q})|^2}{\omega_j(\mathbf{q})} (1 + n(\omega)) \delta(\mathbf{Q} - \mathbf{q} - \boldsymbol{\tau}) \delta(\omega - \omega_j(\mathbf{q}))$$

← Structure (polarization) factor



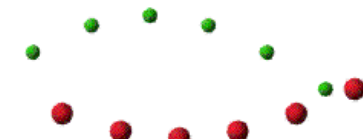
# More complicated structures



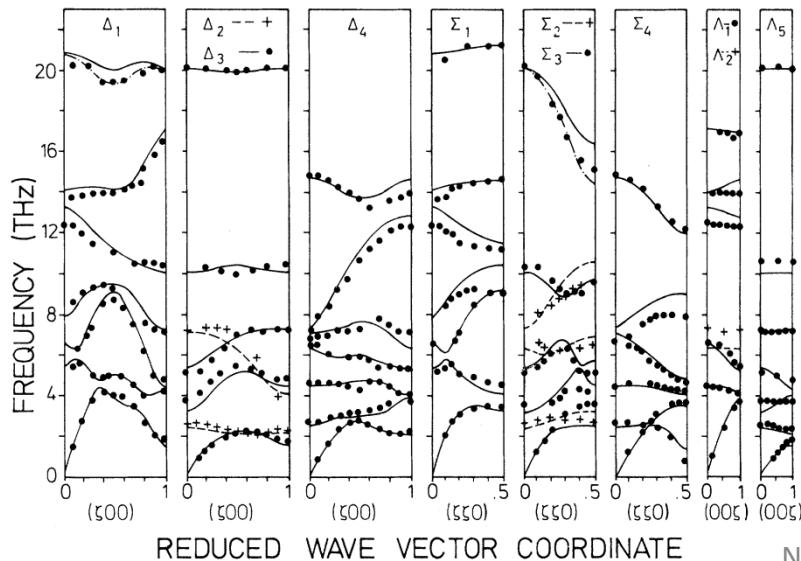
Woods, et al., *Phys. Rev.* **131**, 1025 (1963).



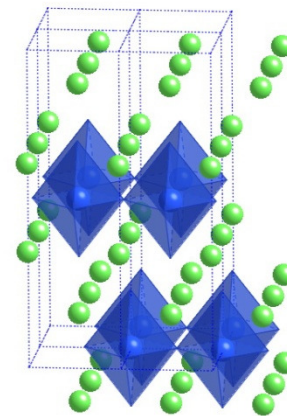
Acoustic phonon



Optical phonon



Chaplot, et al., *Phys. Rev. B* **52**, 7230(1995).

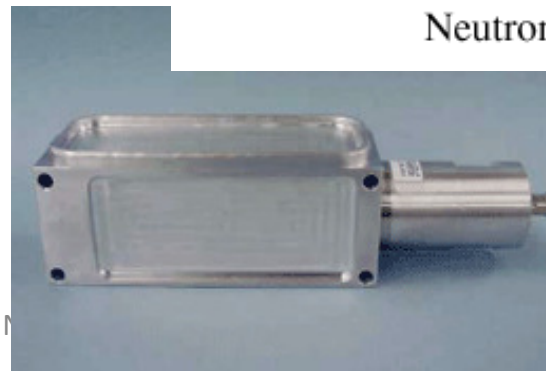
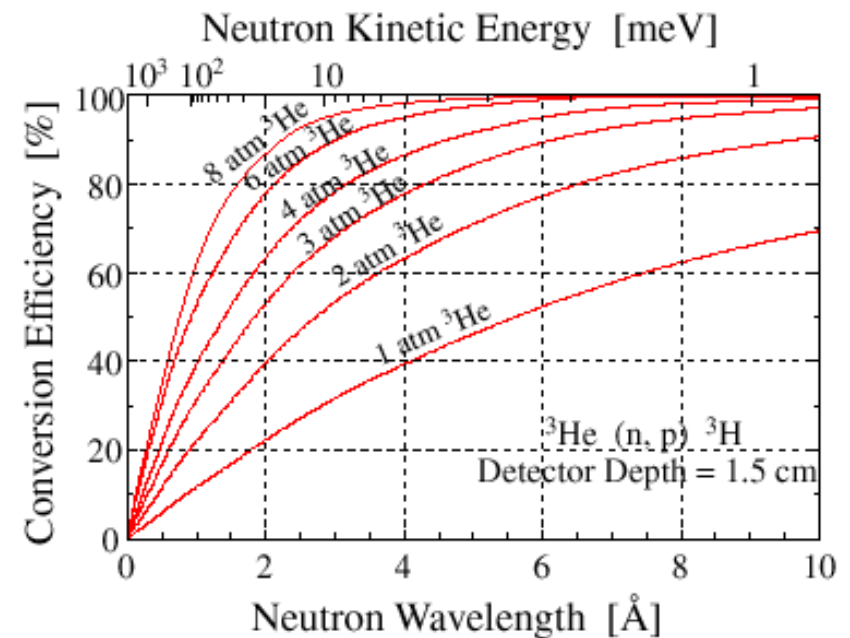


La2CuO4

# Detectors

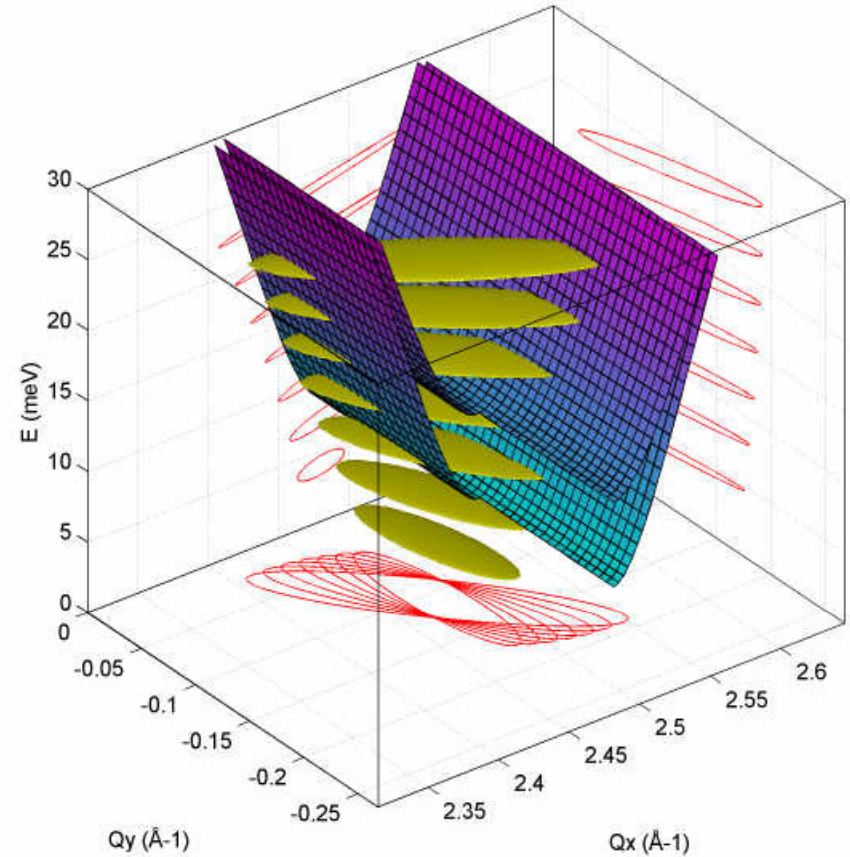


- **Gas Detectors**
  - $n + {}^3\text{He} \rightarrow {}^3\text{H} + p + 0.764 \text{ MeV}$
  - Ionization of gas
  - $e^-$  drift to high voltage anode
  - High efficiency
- 
- **Beam monitors**
  - Low efficiency detectors for measuring beam flux



# Resolution

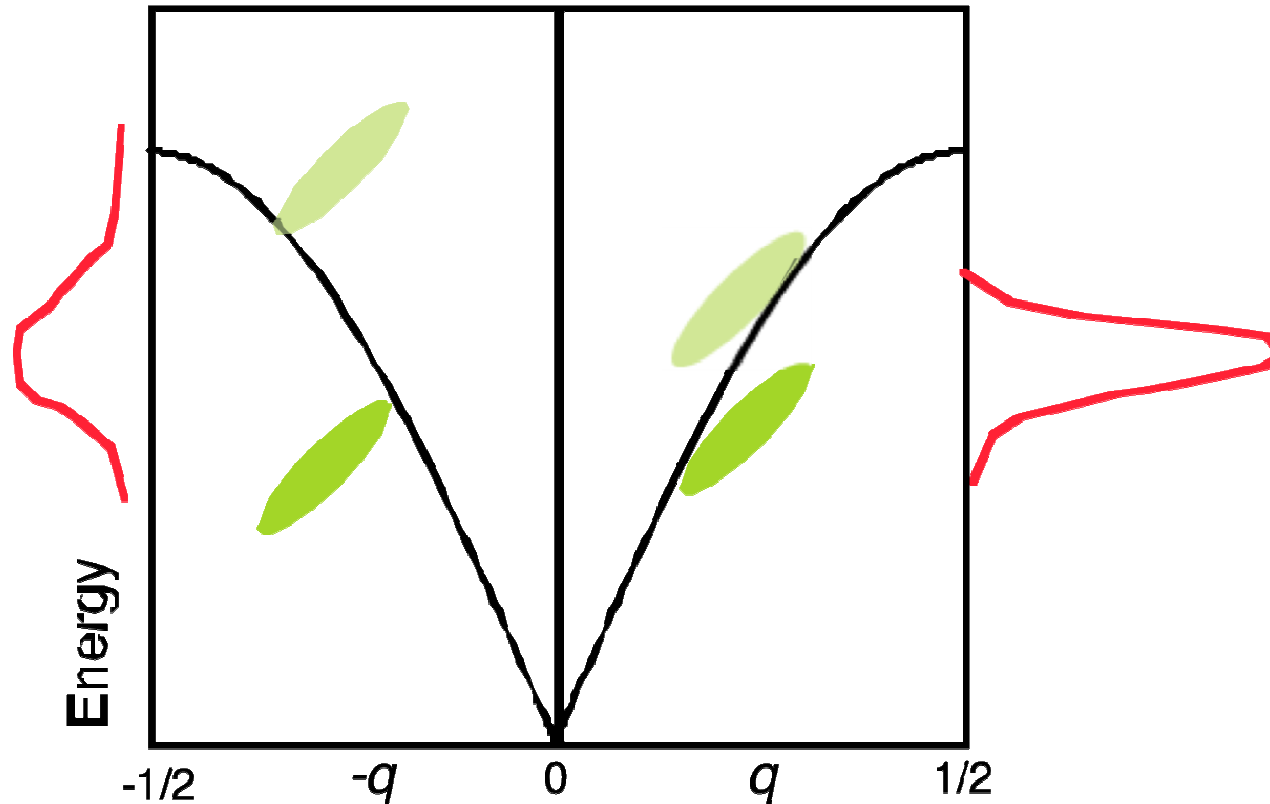
- **Resolution ellipsoid**
  - Beam divergences
  - Collimations/distances
  - Crystal mosaics/sizes/angles
- **Resolution convolutions**



$$I(\mathbf{Q}_0, \omega_0) = \int S(\mathbf{Q}_0, \omega_0) R(\mathbf{Q} - \mathbf{Q}_0, \omega - \omega_0) d\mathbf{Q} d\omega$$

# Resolution focusing

- Optimizing peak intensity
- Match slope of resolution to dispersion



# References

## General neutron scattering

G. Squires, “Intro to theory of thermal neutron scattering”, Dover, 1978.

S. Lovesey, “Theory of neutron scattering from condensed matter”, Oxford, 1984.

R. Pynn, <http://www.mrl.ucsb.edu/~pynn/>.

## Polarized neutron scattering

Moon, Koehler, Riste, Phys. Rev **181**, 920 (1969).

## Triple-axis techniques

Shirane, Shapiro, Tranquada, “Neutron scattering with a triple-axis spectrometer”, Cambridge, 2002.

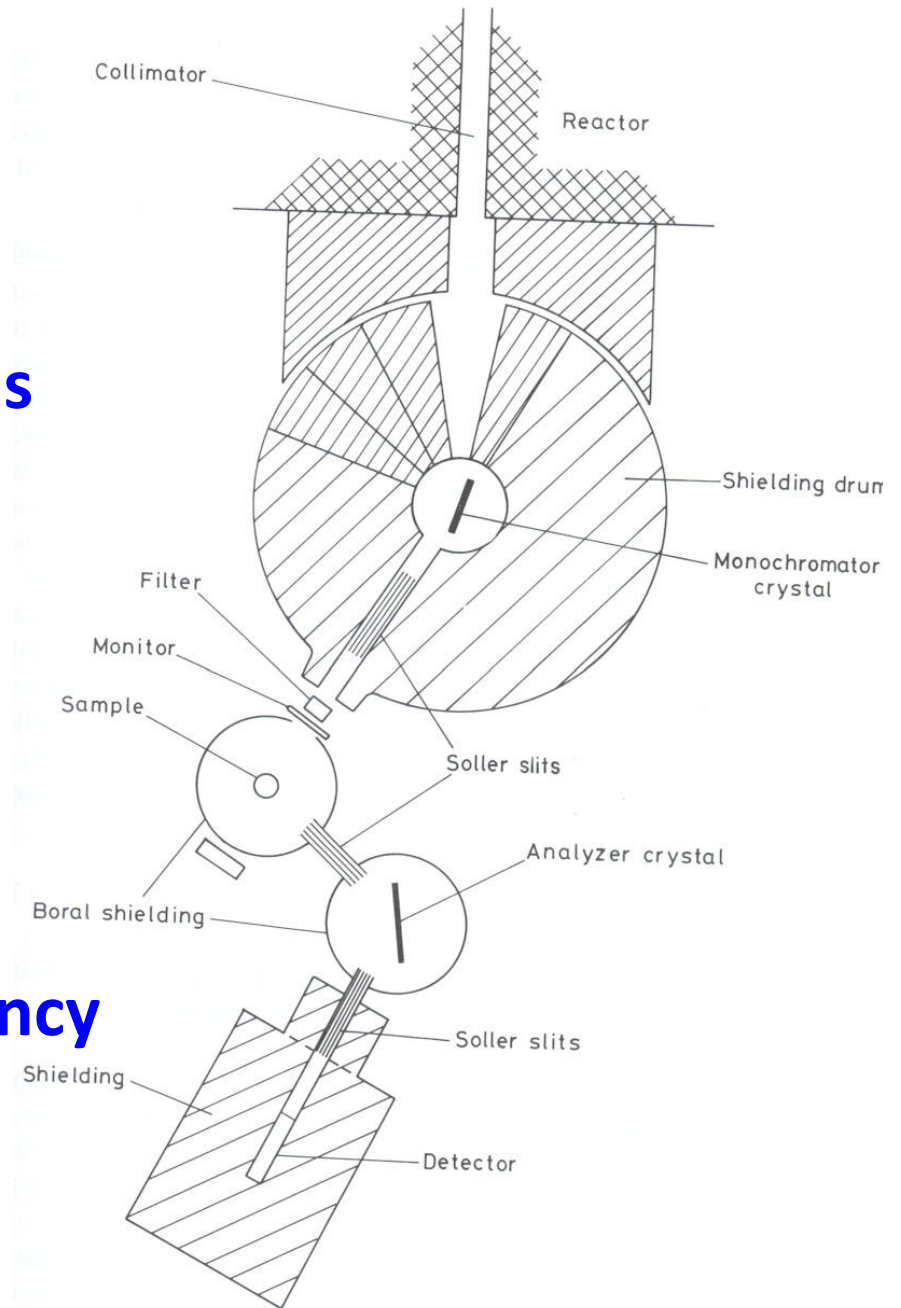
## Time-of-flight techniques

B. Fultz, [http://www.cacr.caltech.edu/projects/danse/ARCS\\_Book\\_16x.pdf](http://www.cacr.caltech.edu/projects/danse/ARCS_Book_16x.pdf)



## Constant $k_f$ :

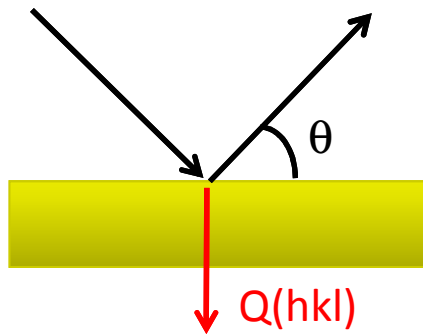
- $k_f$ ,  $\theta_A$  do not change; therefore analyser “efficiency” is constant
- $k_i$ ,  $\theta_M$  do change, but monitor detector normalizes to incident neutron flux
- Monitor detector (low) efficiency goes like  $\sim 1/v \sim 1/k_i$



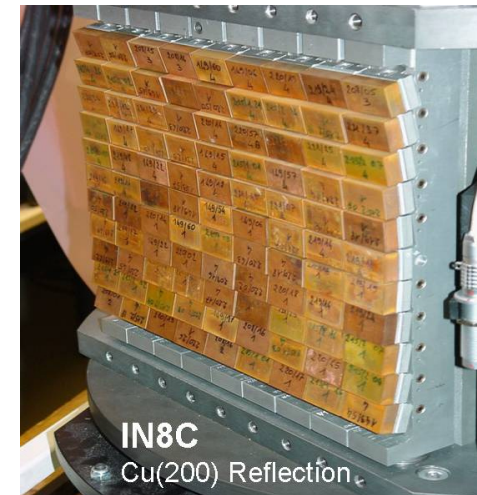


# Monochromators

- Selects the incident wavevector



$$Q(hkl) = \frac{2\pi}{d(hkl)} = 2k_i \sin \theta$$

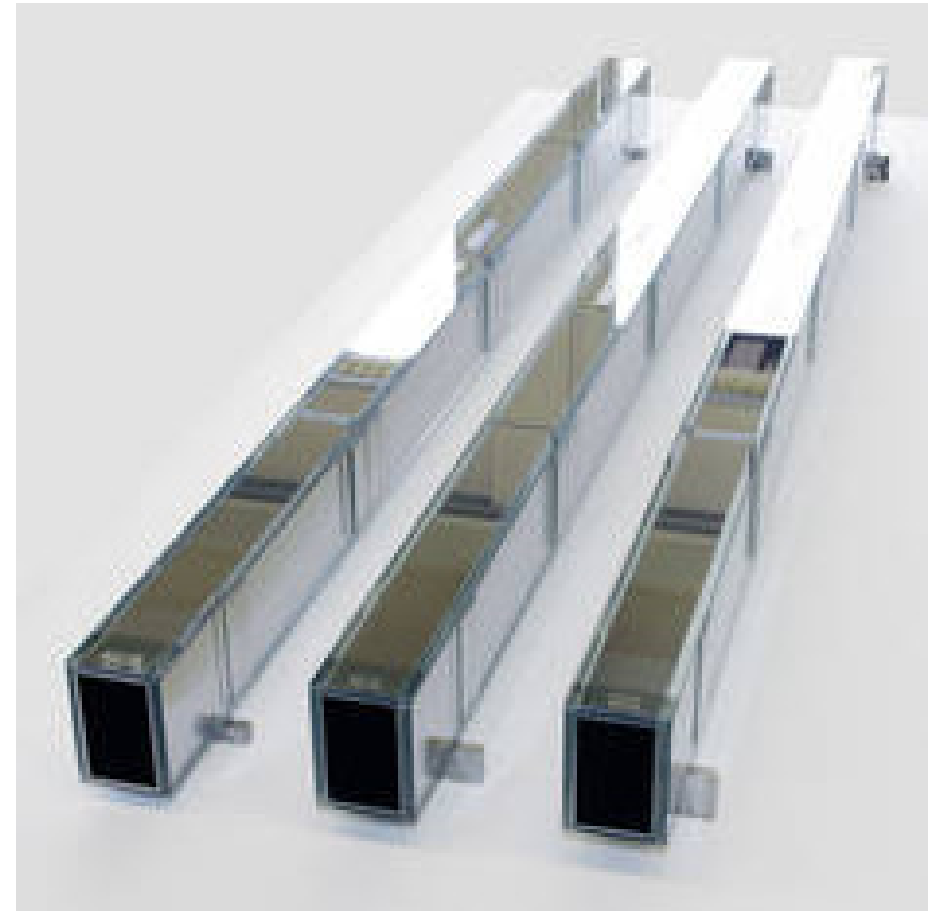


- Reflectivity
- focusing
- high-order contamination  
eg.  $\lambda/2$  PG(004)

Mono	d(hkl)	uses
PG(002)	3.353	General
Be(002)	1.790	High $k_i$
Si(111)	3.135	No $\lambda/2$

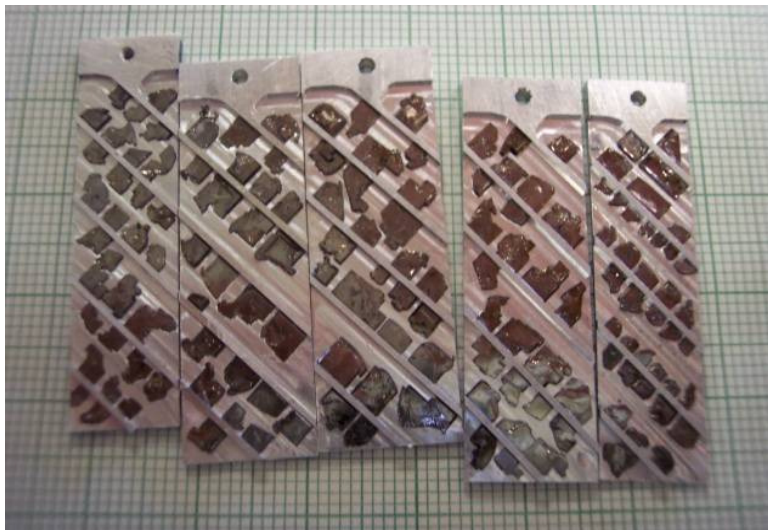
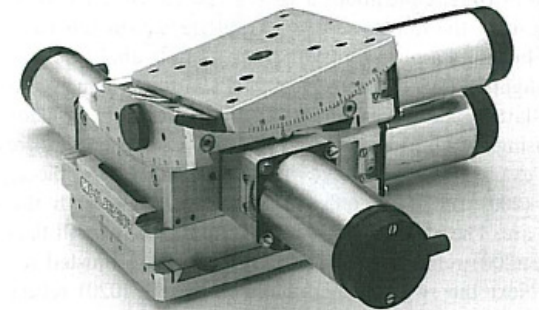
# Guides

- Transport beam over long distances
- Background reduction
- Total external reflection
  - Ni coated glass
  - Ni/Ti multilayers (supermirror)



# Samples

- **Samples need to be BIG**
  - ~ gram or cc
  - Counting times are long (mins/pt)
- **Sample rotation**
- **Sample tilt**



Co-aligned  $\text{CaFe}_2\text{As}_2$  crystals

